[redacted]

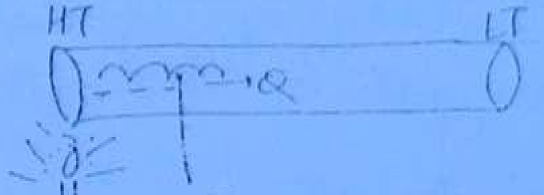
Modes of Heat Transfer.

i) Conduction
ii) Convection
iii) Radiation.

(3)

* Conduction

Molecular lattice vibrations energy transfer (30%)



$k = 2300$, Free electron transfer (70%)

Diamond — Bad conductor of electricity

Silver, Copper, Aluminium, Steels

$k = 405$ W/mK 385 200 15 to 35

** Perfect crystalline lattice arrangement is there in diamond so that k is high. **

$k = 0.2$ W/mK

Insulators ⇒ Asbestos, Rockwool, Glasswool, Refractory brick, Saw dust.

Glass ⇒ Amorphous Material ($k = 1.2$ W/mK)

Defn ⇒ It is the mode of heat transfer which generally occurs in solids due to temperature difference by molecular lattice vibrational energy transfer (around 30%) and also by free electron transfer (around 70%)

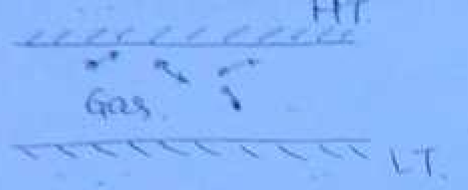
All good electrical conductors are also good conductors of heat due to the presence of free

electrons. e.g. Silver, Copper, Aluminium, steel.
The only exception being Diamond b'coz of its perfect crystalline order of lattice arrangement of molecules.

Gases are very bad conductor of heat.
$K_{air} \Rightarrow 0.026\ W/mK$

(4)

Conduction in gases

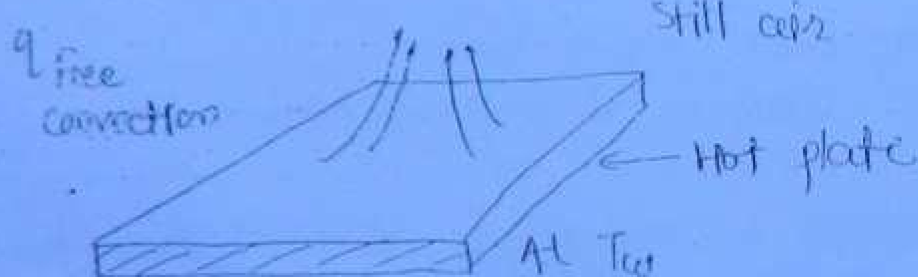


Conduction also occurs in liquids.

$\bar{c} \propto \sqrt{T}$

Conduction also occurs in gases by molecular momentum transfer when high velocity high temp. molecules collide with the low velocity, low temp. molecules. (It is just similar phenomenon as viscosity)
– As the temp. of gases increase, their thermal conductivity also increase.

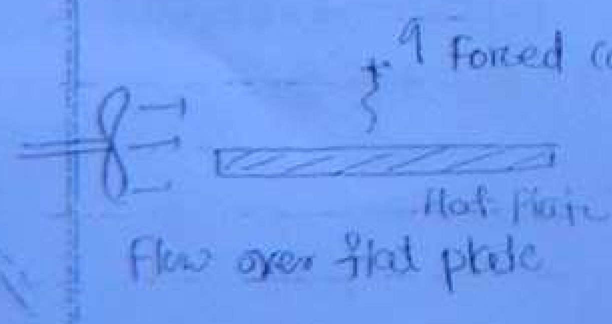
Convection :–



$PV = mRT$

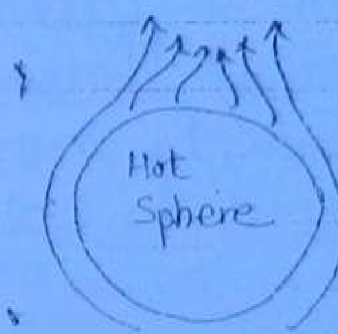
$\frac{P}{\rho} = RT$

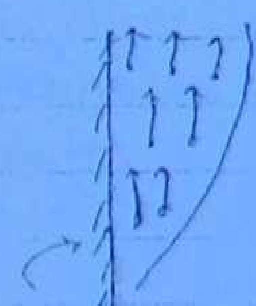
$T \uparrow \rightarrow \rho \downarrow$



Flow over flat plate

convection is made of heat transfer which occurs b/w a hot solid surface & the surrounding cold fluid due to temp. difference associated with the macroscopic bulk displacement of the fluid over the solid surface which is provided by density changes and the resulting buoyancy forces in the case of free convection. or which is provided by an external agency like fan in the case of forced convection.





Hot vertical Wall

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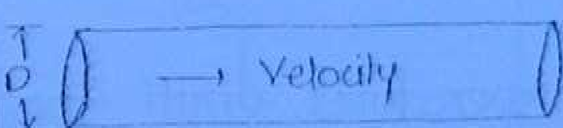
Flow inside ducts

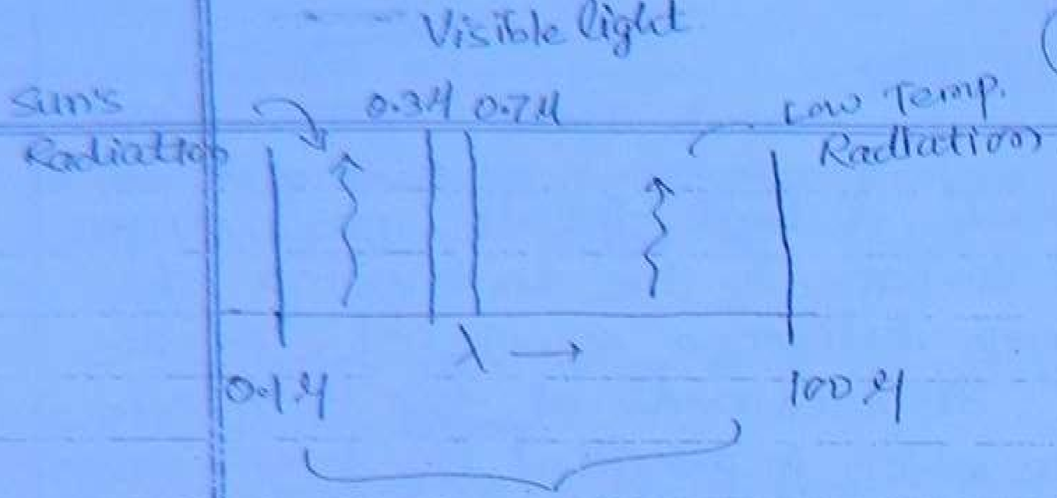


Hot Wall

Babcock Wilcox Boiler - free convection.

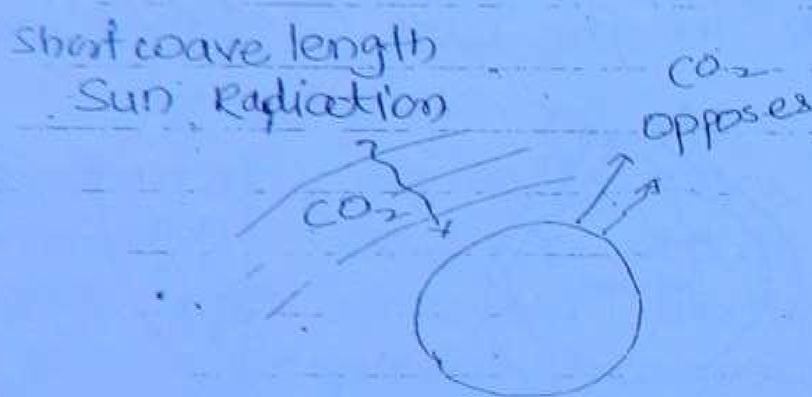
* Radiation.

Electromagnetic waves



Thermal Radiation Spectrum.

(Global Warming)



(6)

Def^n ⇒ Radiation is the mode of heat transfer which does not require any material medium & hence occurs by electromagnetic waves travelling with speed of light.

All bodies at all temperatures emitt thermal radiation except the body at 0 K. Also the rate of emission from any surface of a body per unit time and per unit area is directly directly proportional to the fourth power of absolute temp. of body. ⇒ (Stephen Boltzman Law)*

If the temp. diff. is quite high, thermal radiation completely predominates over conduction & convection

e.g. heat transfer b/w hot flue gases &

Benson boiler we ... ... to create critical pressure of steam i-e. 221 bar.

Date
Page 4

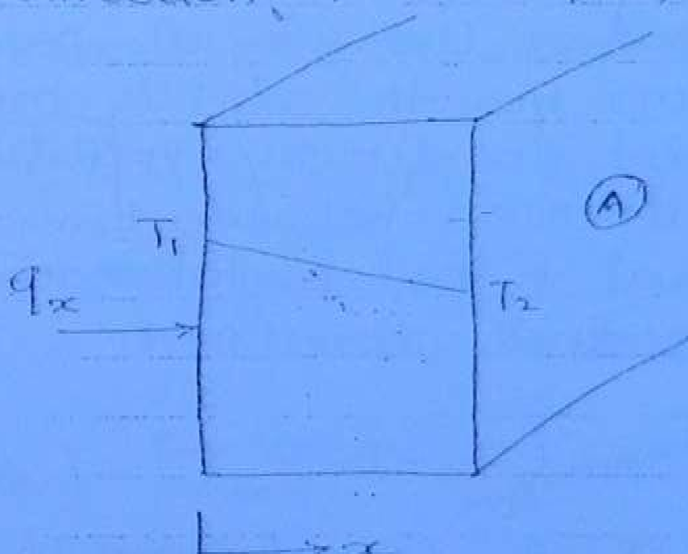
The refractory brick wall in a large furnace of a steam generator/boiler is predominantly by radiation **

+ Laws of Heat Transfer

i) Fourier's Law of Conduction :-

The law states that, "the rate of heat transfer by conduction in any given direction is directly proportional to the temperature gradient along that direction & is also directly proportional to the ~~heat~~ area of heat transfer lying $\perp^{ar}$ to the direction of heat transfer."

(8)



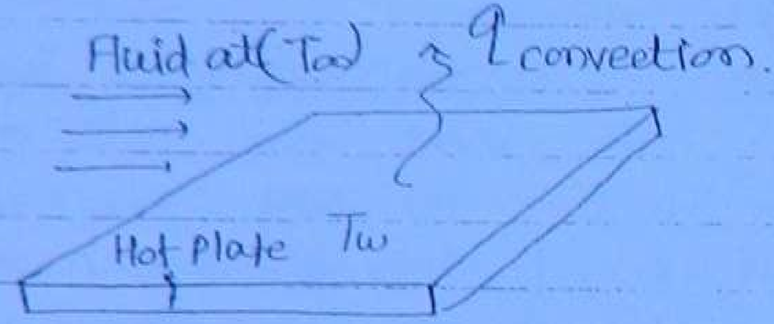
$$q_x \text{ (Rate of H.T.)} \propto -\left(\frac{dT}{dx}\right)$$

$$\propto A$$

$$\boxed{q_x = -KA\left(\frac{dT}{dx}\right)}$$ Joules/sec or Watts.

K = Thermal conductivity
(A property of material of the slab)

2] Newton's Law of Cooling
(For convection H.T.)



(8)

T_w = Temp. of wall of plate

Law states that - The rate of heat transfer by convection b/w the solid & the surrounding cold fluid is directly proportional to the temperature difference between them & is also directly proportional to the area of exposure of the body with the fluid.

$$q_{convection} \propto (T_w - T_\infty)$$

$$\propto A \quad \text{(Area of contact b/w solid \& the fluid)}$$

$$q_{conv.} = h A \Delta T$$

$$= h A (T_w - T_\infty) \text{ Watts.}$$

h = convection heat transfer coefficient in watts/m^2K

[h isn't property of mtrl. but depends on ρ, μ, C_p, K]

Unlike K, h is not property of material but depends upon some of the thermo-physical properties of fluid like density (ρ), specific heat (C_p), dynamic viscosity (μ) & thermal conductivity.

ρ = kg/m^3 C_p = J/kg·K, μ = kg/m·s or Pascal Second
&
K = W/mK

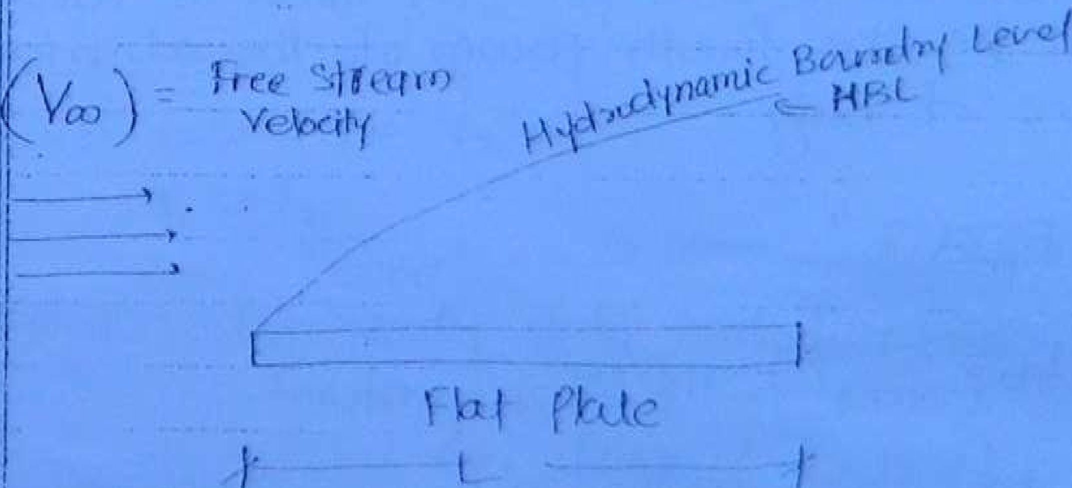
(9)

In forced convection heat transfer

$$h = f(\vec{V}, D, \rho, \mu, C_p, K)$$

$\vec{V}$ = Velocity of fluid
D = characteristic dimension of solid body.

(V_∞) = Free Stream Velocity



or natural

In free convection H.T.

$$h = f(g, \beta, \Delta T, L, \underbrace{\mu, \rho, c_p, k}_{\text{Properties of fluid}})$$

β = Isobaric volume expansion coeff.

$$= \left(\frac{\partial V}{\partial T}\right)_{P=const} \frac{1}{V} = \frac{1}{V}\left[\frac{\Delta V}{\Delta T}\right]_{p=const.}$$

unit is per kelvin /K. (10)

For Ideal gas like air $\beta = \frac{1}{T}$

~~8/1 Sept 2018~~

37 Stefan - Boltzman Law of Radiation.

The law states that - The total radiation energy emitted from the surface of a black body per unit time and per unit area is directly proportional to the fourth power of the absolute temperature of body.

$$E_b \propto T^4$$

$$E_b = \sigma T^4 \quad W/m^2 = \frac{J}{sec.m^2}$$

where σ = Stefan-Boltzman const.

$= 5.67 \times 10^{-8} \; W/m^2K^4$

A black body is the body which absorbs all the incident radiation falling upon it. It is an also an ideal emitter.

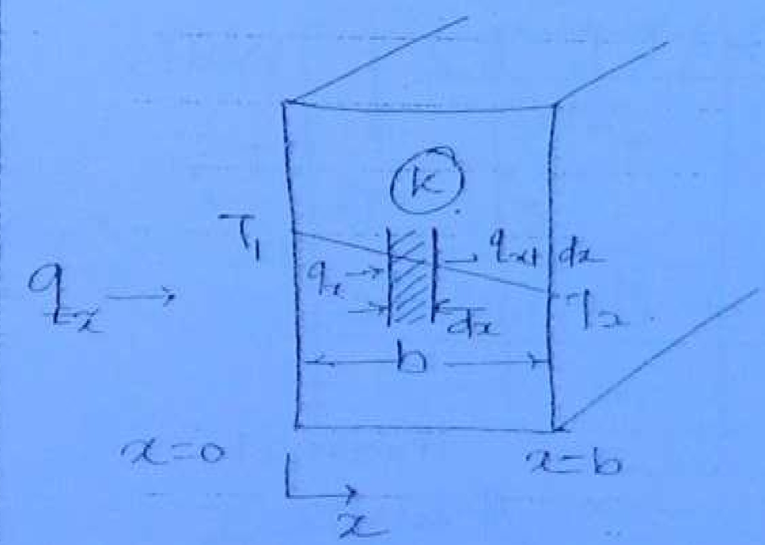
A thermally black body need not appear black in colour to the human eye.

e.g. Snow & Ice. Any good absorber is always a good emitter (Kirchoff's law)

*

(11)

* Conduction Heat Transfer through a slab.



At $x=0 \Rightarrow T=T_1$

At $x=b \Rightarrow T=T_2$

$T_1 > T_2$

Assumptions :- (i) Steady state heat transfer (Temp. isn't f^n of time)

i.e. Temp $\neq f$(time)

(ii) One dimensional conduction.

(iii) Uniform thermal conductivity (k).

$k \neq f(x) \neq f(\text{temp})$

$$q_x = -kA\frac{dT}{dx}$$

$$\int_0^b q_x \cdot dx = \int_{T_1}^{T_2} -kA\,dT$$

$$q_x = q_{x+dx} \qquad (12)$$

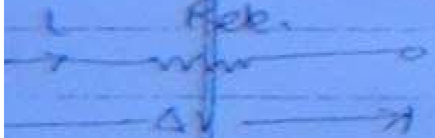
$$q_x = C \neq f(x)$$

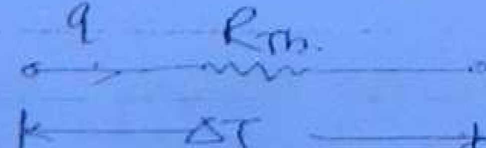
$$\therefore \; q_x \times (b) = kA(T_1 - T_2)$$

$$\therefore \; q_x = \left[\frac{kA(T_1 - T_2)}{b}\right] \text{ Watts}$$

Electrical Analogy of Conductance

Electrical		Thermal
i (Amp)	$\rightarrow$	q watts
ΔV (emf)	$\rightarrow$	ΔT (°C) $\Rightarrow$ Thermal Potential
R_{ele} (Ohm)	$\rightarrow$	R_{Th}.





$$R_{ele} = \left(\frac{\Delta V}{i}\right)\Omega \qquad R_{Th} = \left(\frac{\Delta T}{i}\right) °C/\text{Watt}$$

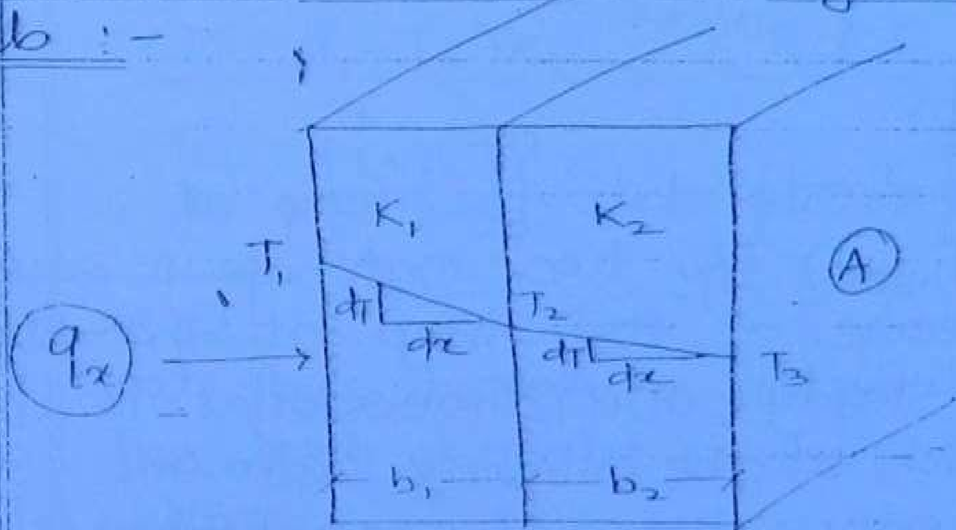
or K/Watt

$$R_{(Th)\,(Slab)} = \frac{b}{KA} = \left(\frac{T_1 - T_2}{q}\right) \quad (13)$$

b = width of slab (Higher the width, more the resistance lower the H.T. rate)

Lower the k value, more the Resistance (R_{Th}).

* <u>Conduction heat transfer through a composite slab</u> :-

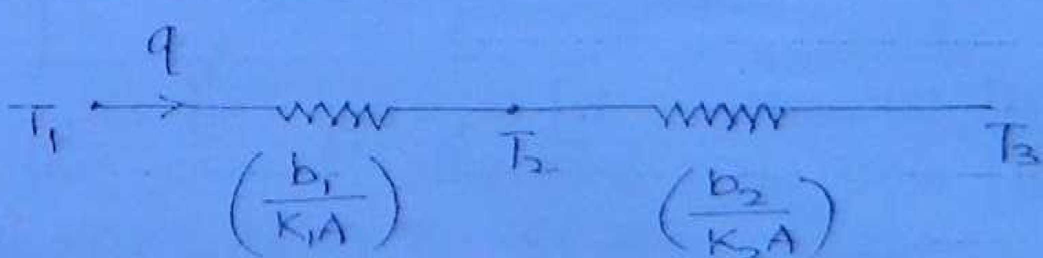


Let ⇒
Steady State
One dimensional
Uniform k value

$$T_1 > T_2 > T_3$$

T_2 = Junction / Interface temperature °c

Thermal circuit :-



$$q = \frac{(T_1 - T_3)}{(\Sigma R_{Th})} = \left[\frac{T_1 - T_3}{b_1/K_1A + b_2/K_2A}\right] \text{ Watts}$$

Heat transfer rate per unit area

$$= \text{Heat flux} = \frac{q}{A} = \left[\frac{T_1 - T_3}{b_1/k_1 + b_2/k_2}\right] \text{W/m}^2$$

$$q_x = -kA\left(\frac{dT}{dx}\right)$$

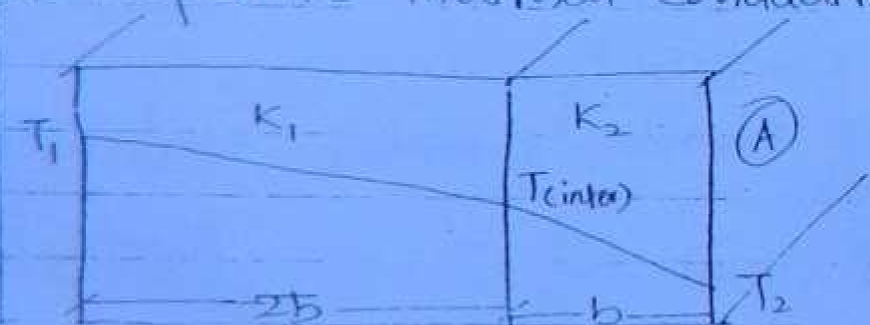
(14)

since $\left(\frac{dT}{dx}\right)_{in①} > \left(\frac{dT}{dx}\right)_{in②}$

$$\Rightarrow k_2 > k_1$$

$\boxed{\text{GATE 2006}}$

In a composite slab the temp. at a inter-face (T_{inter}) b/w two materials is equal to the ~~the~~ average of the temp. at the two ends. Assuming steady, one dimensional heat conduction, which of the following statement is true are about the respective thermal conductivities



(A)

a] $2k_1 = k_2$ b] $k_1 = k_2$ c] $2k_1 = 3k_2$

d] $k_1 = 2k_2$

$$T_1 \xrightarrow{q} \underset{\frac{2b}{K_1 A}}{\text{wwww}} \; T_{inter} \; \underset{\frac{b}{K_2 A}}{\text{wwww}} \; T_2$$

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$$q = \frac{T_1 - T_{inter}}{2b/K_1 A} = \frac{T_{inter} - T_2}{b/K_2 A}$$

Given $T_{inter} = \left(\frac{T_1 + T_2}{2}\right)$

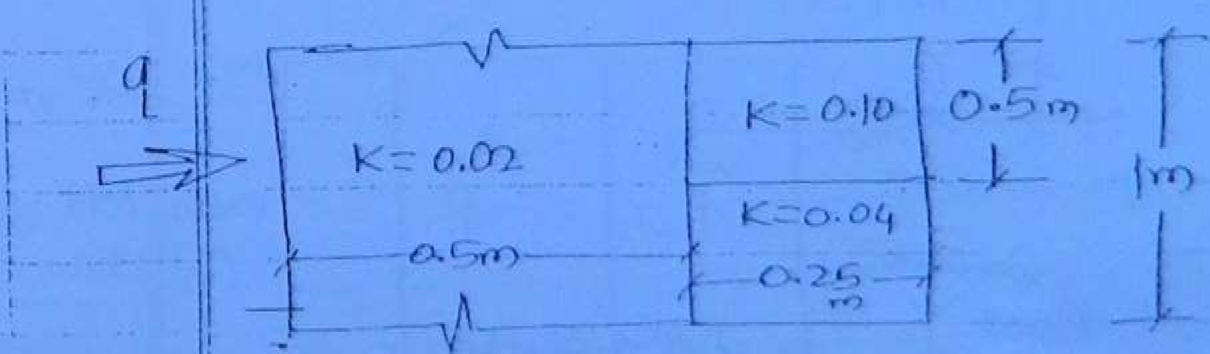
$$\cancel{\frac{(T_1 - (T_1 - T_2))K_1}{4} = \frac{(T_1 + T_2 - T_2)K_2}{2}}$$

$$\left\{T_1 - \left[\frac{T_1 + T_2}{2}\right]\right\}\frac{K_1}{2} = \left[\left(\frac{T_1 + T_2}{2}\right) - T_2\right]K_2$$

$$\left(\frac{T_1 - T_2}{2}\right)\frac{K_1}{2} = \left(\frac{T_1 - T_2}{2}\right)K_2 \quad \text{i.e. } \boxed{K_1 = 2K_2}$$

$\boxed{\text{GATE 2005}}$

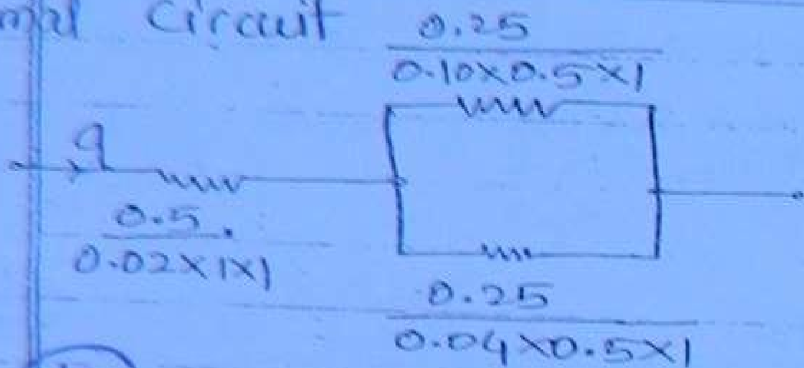
Heat flows through a composite as shown below. The depth of slab is 1 m. The k values are in W/m.k



The overall thermal resistance

a] 17.2 b] 21.9 c] 28.6 d] 39.2

Thermal Circuit



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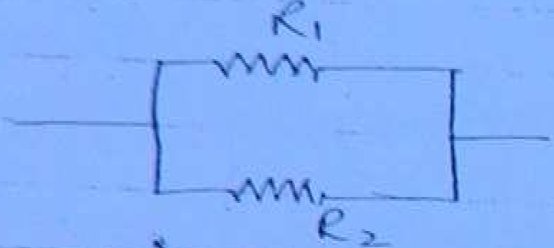


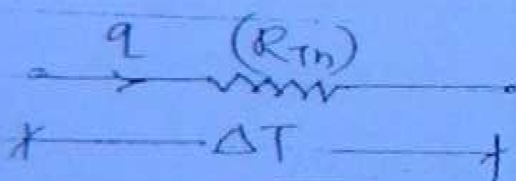
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

Answer is C 28.6

Convection Thermal Resistance

From Newton's law of cooling

$$q = hA\Delta T$$



$$\therefore R_{Th} = \left(\frac{\Delta T}{q}\right) {}^\circ C/\text{Watt}$$

$$= \left(\frac{1}{hA}\right)$$

A = Area of contact between solid & fluid.

[Rate of H.T. by water is 25 times more than that of air.]

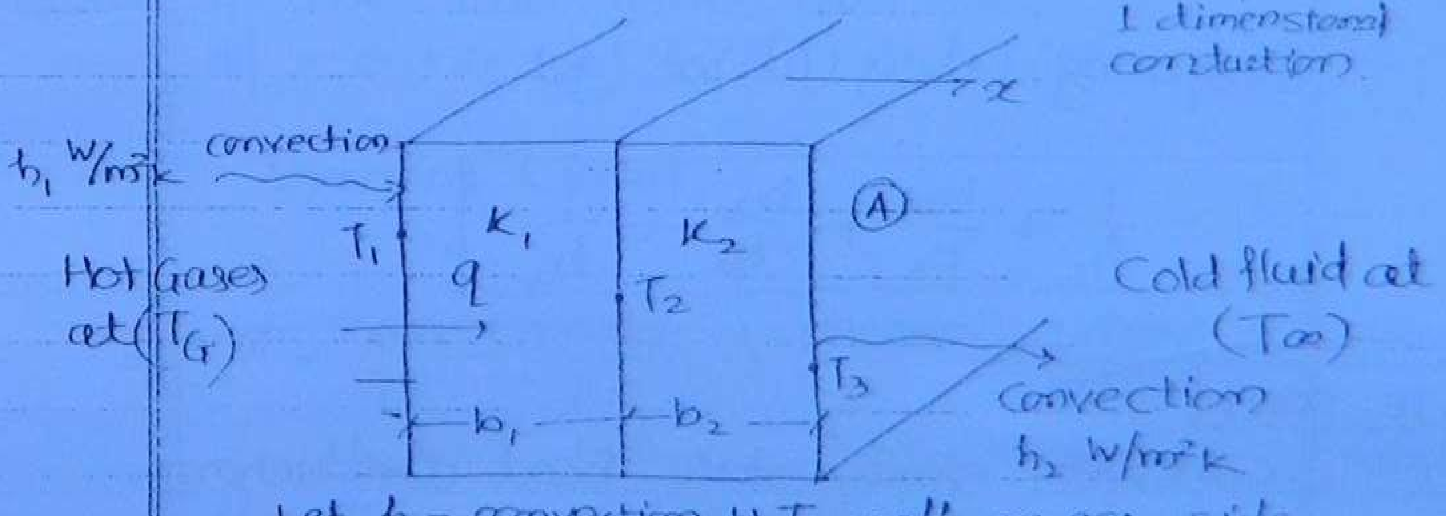
Ranges of h

i) Free convection in gases $\Rightarrow$ 3 to 25 W/m^2K
ii) Forced — " — $\Rightarrow$ 25 to 450 W/m^2K
iii) Free convection in liquids like water $\Rightarrow$ 50 to 600 W/m^2K
iv) Forced — " — $\Rightarrow$ 500 to 3000 W/m^2K
v) Condensation H.T. $\Rightarrow$ 3000 to 25000 W/m^2K
vi) Boiling H.T. $\Rightarrow$ 5000 - 50000 W/m^2K

[Condensation $\Rightarrow$ change of phase of vapour to liquid
Boiling $\Rightarrow$ liquid to vapour]

* Conduction - convection H.T. through a composite ~~cylinder~~ Slab

Let, steady state 1 dimensional conduction



Let h_1 = convection H.T. coeff. on gas side
h_2 = — " — cold fluid side

$$\left(\frac{1}{h_1 A}\right) \quad \left(\frac{b_1}{K_1 A}\right) \quad \left(\frac{b_2}{K_2 A}\right) \quad \left(\frac{1}{h_2 A}\right)$$

$$\therefore q = \frac{T_G - T_\infty}{\sum R_{th}}$$

$$(18) = \frac{T_G - T_\infty}{\frac{1}{h_1 A} + \frac{b_1}{k_1 A} + \frac{b_2}{K_2 A} + \frac{1}{h_2 A}}$$

Heat transfer rate per unit area.

$$\text{(Heat flux)} = \frac{q}{A} = \left[\frac{T_G - T_\infty}{\frac{1}{h_1} + \frac{b_1}{k_1} + \frac{b_2}{k_2} + \frac{1}{h_2}} \right] \quad \text{— (i)} \quad \text{watt/m}^2$$

* Overall Heat Transfer Coefficient – (U)

It takes into account all the modes of heat transfer & is defined from equation

$$\boxed{Q = UA\Delta T} \qquad U \Rightarrow W/m^2K \quad \text{— (ii)}$$

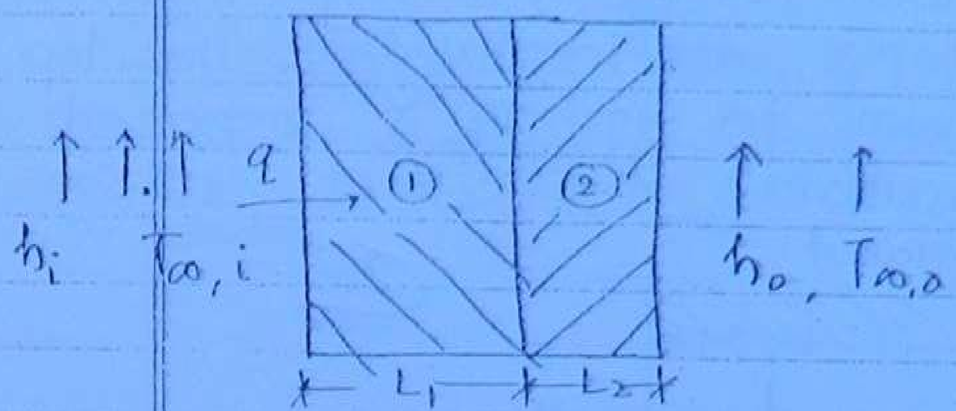
Comparing (i) & (ii) we get,

$$\boxed{\frac{1}{U} = \frac{1}{h_1} + \frac{b_1}{k_1} + \frac{b_2}{K_2} + \frac{1}{h_2}}$$

GATE 2009

Consider steady state heat conduction across the thickness in a plane composite wall (as shown in fig.) exposed to convection conditions on both sides.

(19)



Given $h_i = 20\ W/m^2k$

$h_d = 50\ W/m^2k$

$T_{\infty i} = 20^\circ C$ $\quad T_{\infty o} = -2^\circ C$

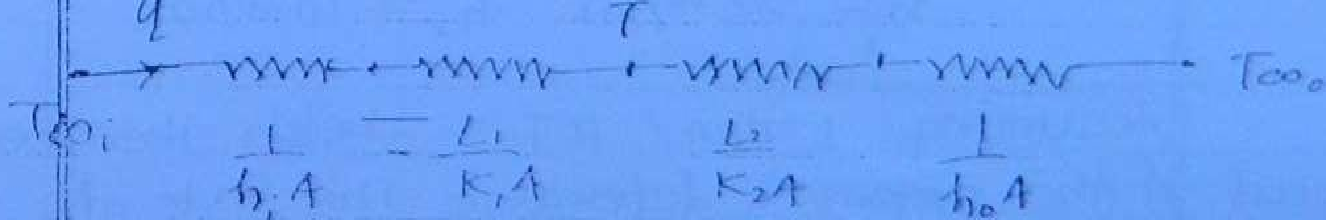
$K_1 = 20\ W/m^2k$ $\quad K_2 = 50\ W/mk$

$L_1 = 0.30\ m$ $\quad L_2 = 0.15\ m$

Assuming negligible contact resistance b/w wall surfaces, the interface temperature T in °C ~~after~~ of the two walls will be

a] -0.50 b] 2.75 c] 3.75 d] 4.50

⇒ Thermal Circuit



$$\therefore q = \frac{T_{\infty i} - T}{\frac{1}{h_i A} + \frac{L_1}{K_1 A}} = \frac{T - T_{\infty,o}}{\frac{L_2}{K_2 A} + \frac{1}{h_o A}}$$

[GATE 2001]

A composite wall having unit length normal to the plane of paper, is insulated at the top and bottom as shown in fig. It is comprised of four different materials A, B, C & D.

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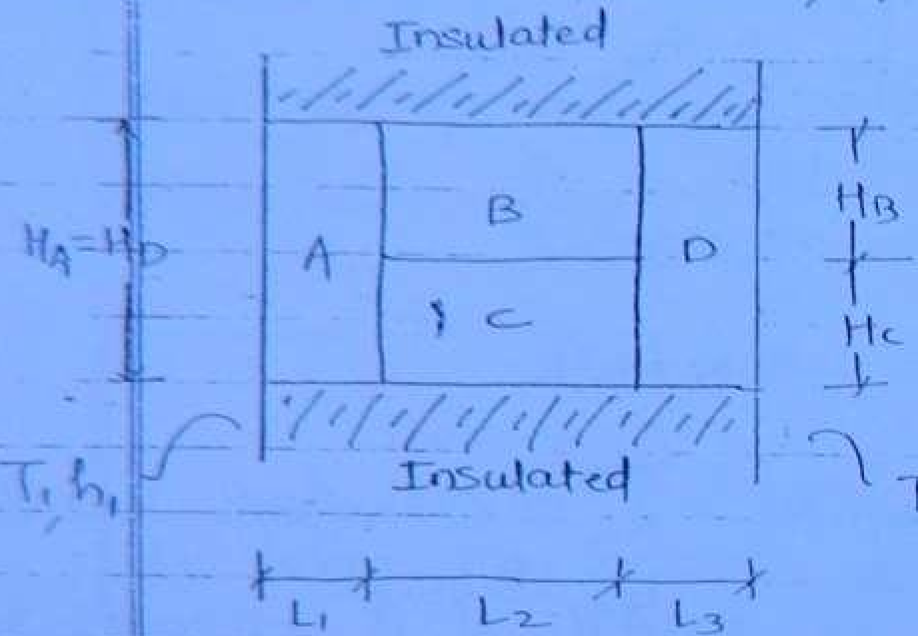


$H_A = H_D = 3m \qquad H_B = H_C = 1.5m$

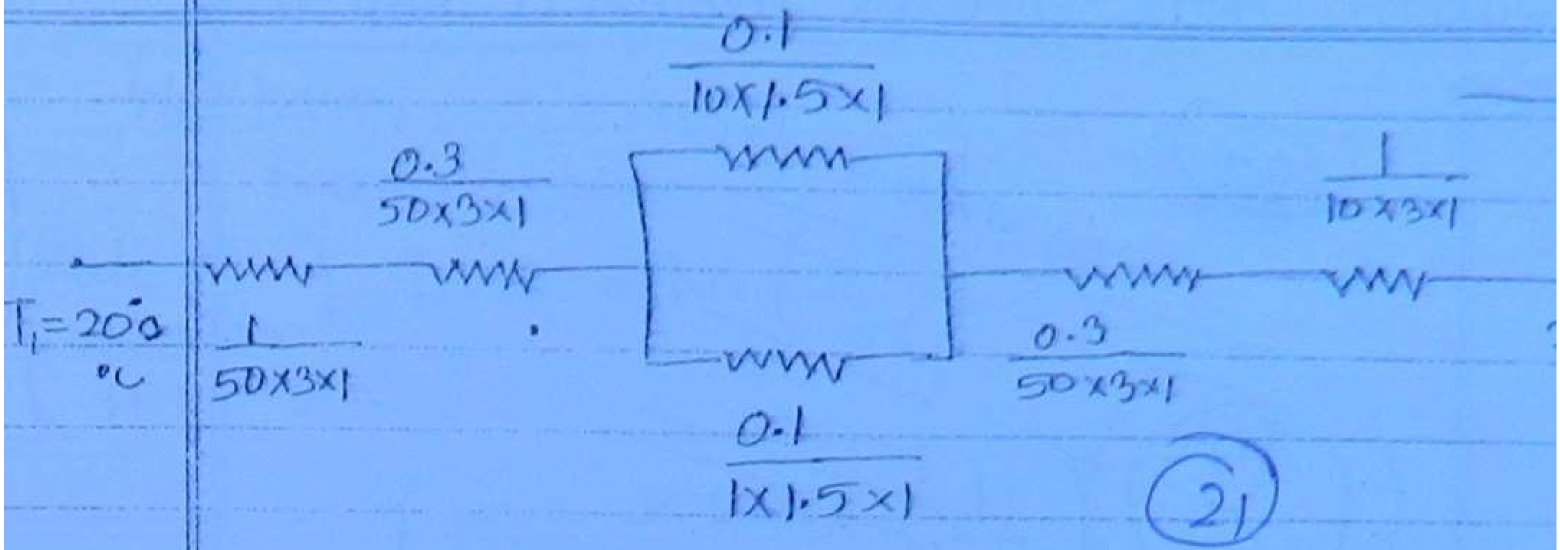
$L_1 = L_3 = 0.3m \qquad L_2 = 0.1m$

$k_A = k_D = 50\ W/mK \qquad k_B = 10\ W/mK$

$k_C = 1\ W/mK$. The fluid temp.s & heat transfer coeff. are. $T_1 = 200°C$, $T_2 = 25°C$

$h_1 = 50\ W/m^2K \quad h_2 = 10\ W/m^2K$.

Assuming 1 dim. H.T. sketch the thermal circuit of the system & determine the rate of H.T. ~~to~~ through the wall.



Text book

Incropera & Dewitt

J.P. Holman

Sukhatme

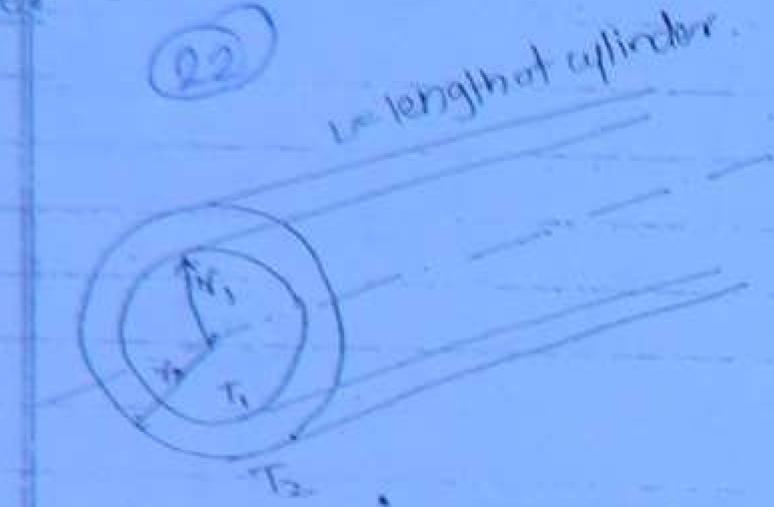
Ozisik

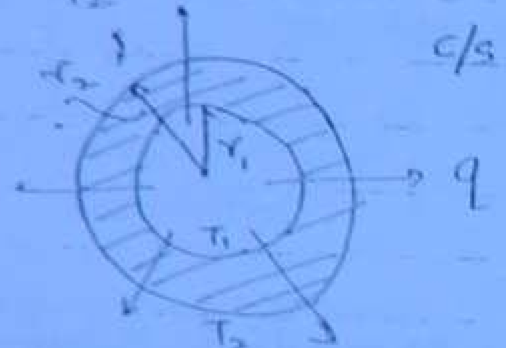
Cengel & Boles

D.S. Kumar → Numerical Problems.

* Conduction Heat Transfer through a hollow cylinder

(22)





Assumptions

(i) Steady state H.T (ii) One dimensional radial conduction H.T. (iii) k is uniform $k \neq f(r)$

At ~~T~~, $r = r_1$ $T = T_1$ $(T_1 > T_2)$

At $r = r_2$ $T = T_2$

Conduction occur radially outwards from inside surface at r_1 & T_1 to outside surface at ~~T_2~~ r_2 & T_2.

Here the conduction area of H.T. is increasing from inside to the outside

At any radius r, area of conduction H.T

$= (2\pi r L)$

We have,

Fourier's Law

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(Rate of conduction H.T.) $q = -KA\left(\frac{dT}{dr}\right)$ Watts.

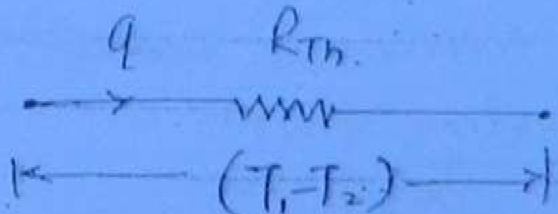
$$q = -k\, 2\pi r L\left(\frac{dT}{dr}\right)$$

$$q \neq f(r)$$

$$\int_{r_1}^{r_2} q \frac{1}{r}\, dr = \int_{T_1}^{T_2} -k\, 2\pi L\, dT$$

$$q = \frac{2\pi k L\,(T_1 - T_2)}{\log_e\left(\frac{r_2}{r_1}\right)}$$

Thermal Circuit



$$(R_{Th})_{cylinder} = \frac{\Delta T}{q} = \frac{\ln(r_2/r_1)}{2\pi k L} \quad K/watt$$

* Conduction of H.T. through a composite cylinder.

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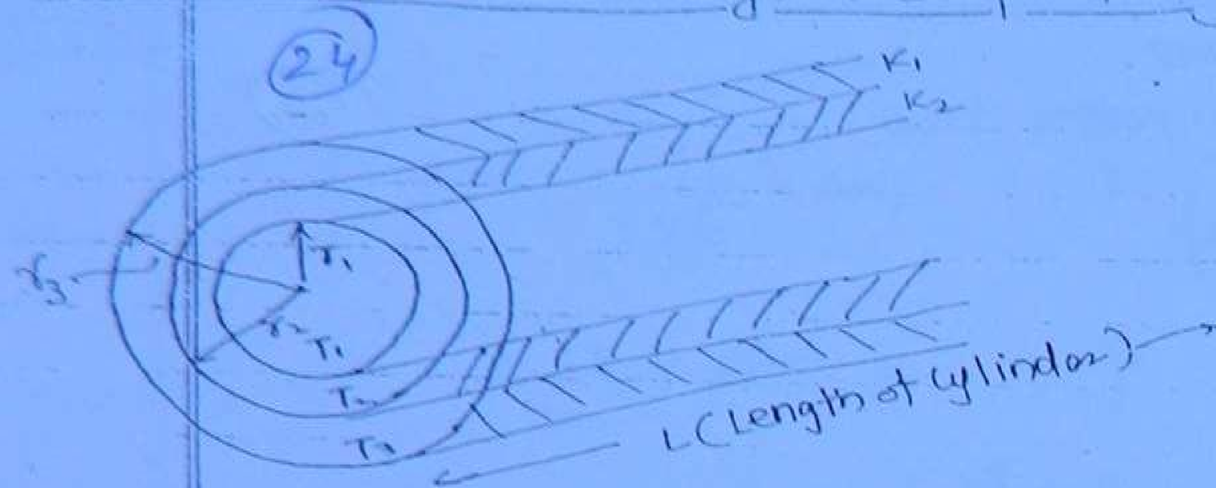


Assumptions

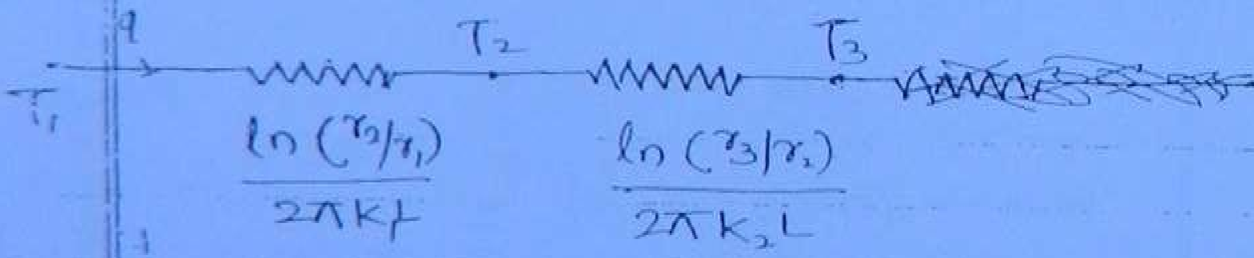
(i) Steady state ii) One dimensional radial conduction H.T. (iii) Uniform value of k.

At $r = r_1$: $T = T_1$, $r = r_2$: $T = T_2$

$r = r_3$: $T = T_3$.

Thermal circuit

Junction/Interface Temp.



(Rate of H.T)

$$q = \frac{(T_1 - T_3)}{\Sigma R_{Th}} \quad \text{Watts}$$

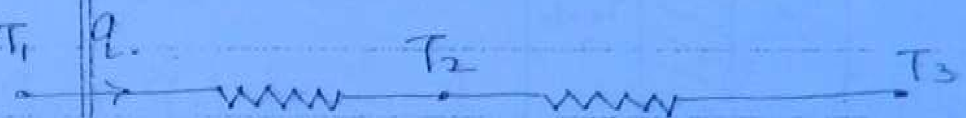
$$q = \frac{T_1 - T_3}{\frac{\ln(r_2/r_1)}{2\pi k_1 L} + \frac{\ln(r_3/r_2)}{2\pi k_2 L}}$$

GATE 2004 (25)

A stainless steel tube $K_S = 19$ W/mK of 2 cm inner dia. & 5 cm outer dia. is insulated with ~~three~~ 3 cm thick asbestos $K_A = 0.2$ W/mK. If the temperature diff. b/w the innermost & outermost surface is 600°C. The heat transfer rate per unit length is

(i) 0.94 W/m (ii) 944 W/m
(iii) 9447.2 W/m (iv) 94.7 W/m

ction



$$\frac{\ln(5/2)}{2\pi \times 19 \times L} \qquad \frac{\ln(5.5/2.5)}{2\pi \times 0.2 \times L}$$

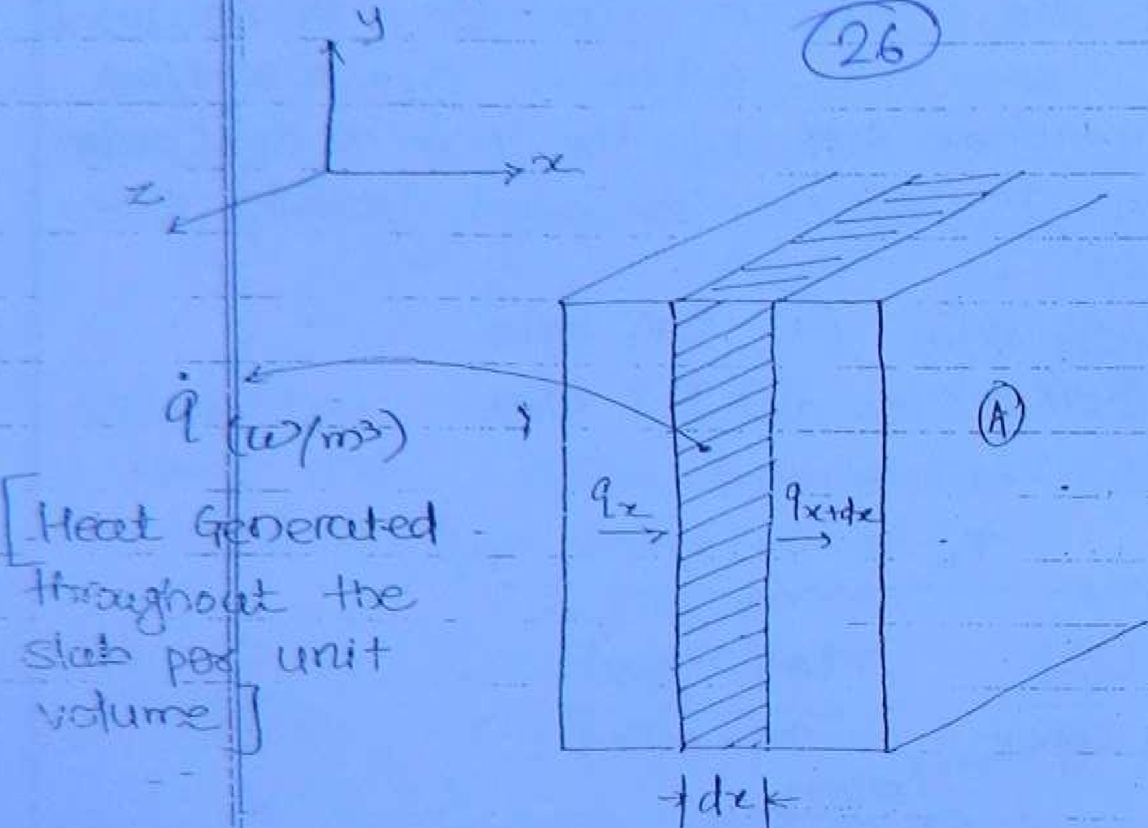
$$\therefore \quad q = \frac{(T_1 - T_3)}{\dfrac{\ln(5/2)}{2\pi \times 19 \times L} + \dfrac{\ln(5.5/2.5)}{2\pi \times 0.2 \times L}}$$

$$\therefore \quad \frac{q}{L} = \frac{600}{\dfrac{\ln(5/2)}{2\pi \times 19} + \dfrac{\ln(5.5/2.5)}{2\pi \times 0.2}}$$

$$=$$

* To derive generalised (3-dimensional, Steady or unsteady, with or without heat generation) conduction equation *

(26)



Consider a differential element in the slab of length dx

$$\dot{q} = f(x, y, z)$$

Let q_x = Heat conducted into the element

$$= -KA\left(\frac{dT}{dx}\right)$$

q_{x+dx} = Heat conducted out of element

$$= q_x + \frac{\partial}{\partial x}(q_x)\,dx$$

$$q_{\text{(Generated in element)}} = \dot{q}(A \cdot dx)$$

Writing the energy balance eqn for element & along the x-direction. i.e. heat conducted into element + heat generated
= Heat conducted out of element + Rate of change of Internal energy of element w.r.t. time

$$q_x + \dot{q} A \cdot dx = q_{x+dx} + \frac{\partial}{\partial \tau}(m C_p T)$$

$$q_x + \dot{q} A \cdot dx = \cancel{q_x} + \frac{\partial}{\partial x}(q_x) \cdot dx + \frac{\partial}{\partial \tau}(m C_p T)$$

$$\dot{q} A\, dx = \frac{\partial}{\partial x} \left(-KA \frac{dT}{dx}\right) dx + \rho A \cdot dx\, C_p \frac{\partial T}{\partial \tau}$$

$$K \frac{\partial^2 T}{\partial x^2} + \dot{q} = \rho C_p \frac{\partial T}{\partial \tau} \quad \cdots \because T = f(x, y, z, \tau)$$

Similarly & simultaneously writing the energy balance eqn to the element for all the 3 directions x, y & z put together we get -

$$K \frac{\partial^2 T}{\partial x^2} + K \frac{\partial^2 T}{\partial y^2} + K \frac{\partial^2 T}{\partial z^2} + \dot{q} = \rho C_p \frac{\partial T}{\partial \tau}$$

$$\boxed{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \left(\frac{\rho C_p}{k}\right)\frac{\partial T}{\partial \tau}}$$

* **Thermal Diffusivity (α)**

A property of material as the ratio b/w thermal conductivity (k) & thermal capacity.

$$\boxed{\alpha = \frac{k}{(\rho C_p)} \quad m^2/sec.}$$

Thermal diffusitivity (α) is the ability of material to allow, the heat energy to get diffused through the medium.

Prandtl Number (for fluids only)

$$P_r = \left(\frac{\nu}{\alpha}\right) \qquad \nu = \text{kinematic viscosity.}$$

If there is no heat generation,

$$\dot{q} = 0$$

If conditions are steady

$$\partial T / \partial \tau = 0$$

We get,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$$

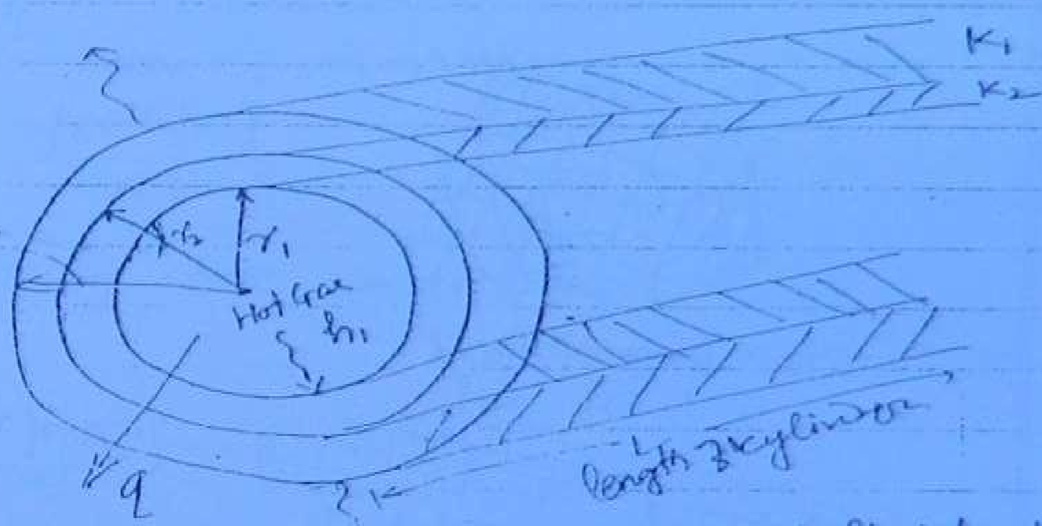
$$\boxed{\nabla^2 T = 0} \quad \text{Laplace eqn in } T$$

... h is not only ... its ... associated with contact material & fluid.

This is the three dimensional heat conduction eqⁿ without heat generation under steady state.

(29)

Q. * Convection - conduction H.T. through a composite cylinder.



Convection Cold fluid at T_∞

Let,

T_G = Temperature of gas

T_∞ = Temperature of outside cold fluid

h_1 = Convection H.T. coeff. on gas side (W/m^2K)

h_2 = Convection —— " —— fluid side (W/m^2K)

Assumption ⟹ ③ Assumption are followed.

Thermal Circuit

$$T_G \; \frac{1}{h_1 A_1} \; \frac{\ln(r_2/r_1)}{2\pi K_1 L} \; \frac{\ln(r_3/r_2)}{2\pi K_2 L} \; \frac{1}{h_2 A_2} \; T_\infty$$

A_1 = Inside convection H.T. area = $2\pi r_1 L$

A_2 = Outside —— " —— = $2\pi r_3 L$

$$\therefore q = \frac{T_a - T_\infty}{\Sigma R_{th}}$$

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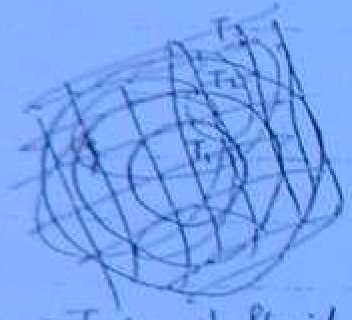
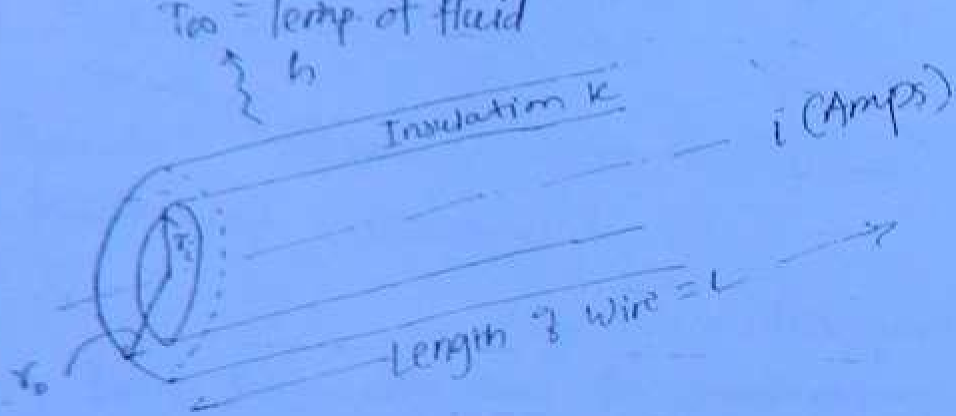
$$\therefore q = \frac{T_a - T_\infty}{\frac{1}{h_1 A_1} + \frac{\ln(r_2/r_1)}{2\pi K_1 L} + \frac{\ln(r_3/r_2)}{2\pi K_2 L} + \frac{1}{h_2 A_2}}$$

* Critical Radius of Insulation





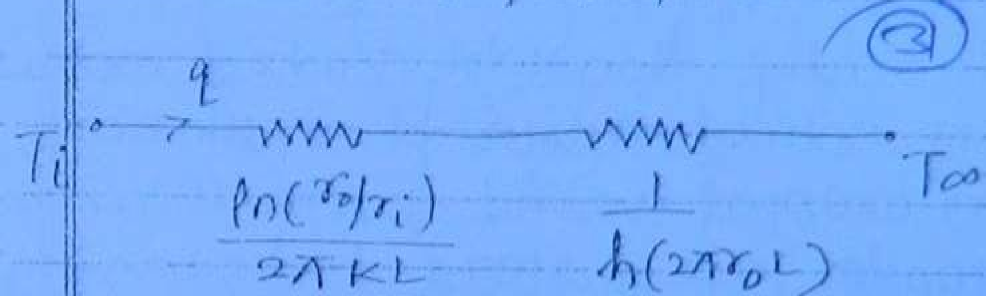
Consider a solid wire of radius r_i inside which there is heat generated due to the passage of electric current.

Let r_0 be the radius at upto which insulation is wrapped & k is the thermal conductivity of insulation.

T_i = Temperature of solid wire at r_i

T_∞ = Temp. of fluid ; h = convection H.T. coeff. b/w surface of, The heat generated in the wire is radially conducted through the insulation & then convected to the surrounding fluid at T_∞ with a convection H-T coeff. of h

Between T_i & T_∞, thermal circuit



Assume steady state conditions.

$$\therefore \quad q = \frac{T_i - T_\infty}{\frac{\ln(r_o/r_i)}{2\pi kL} + \frac{1}{h(2\pi r_o L)}}$$

Rate of heat loss

Keeping all other parameters const; q is function r_o. Greater the insulation wrapped around the wire, higher the value of r_o.

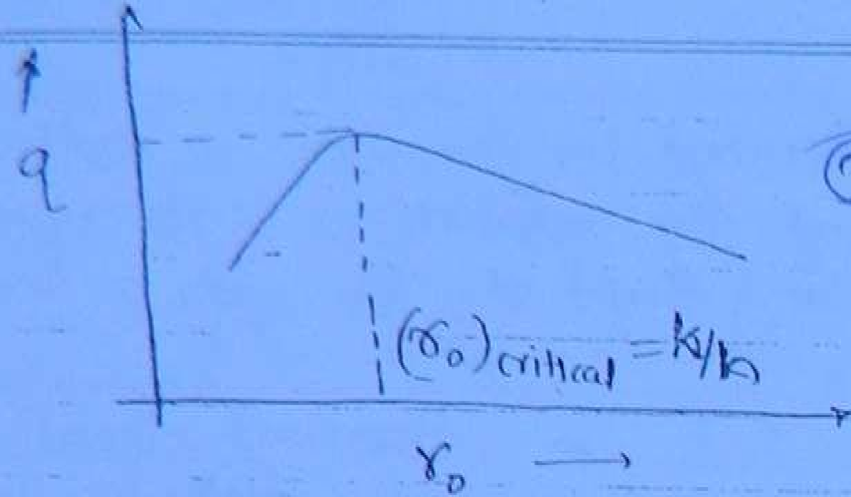
For maximum heat transfer rate q,

$$\frac{dq}{dr_o} = 0$$

$$= \frac{d}{dr_o}\left[\frac{T_i - T_\infty}{\frac{\ln(r_o/r_i)}{2\pi kL} + \frac{1}{h(2\pi r_o L)}}\right] = 0$$

$$\Rightarrow \boxed{r_o = \left(\frac{k}{h}\right)} = \text{Critical Radius of Insulation.}$$

(32)



For sufficiently thin wires whose radius is lesser than critical radius of insulation, any insulation wrapped around it shall increase the heat transfer rate instead of decreasing it. The reason is — there is a rapid decrease in a convection thermal resistance as compared to the increase in conduction thermal resistance. The overall effect being the decrease in total thermal resistance & increase in H.T. rate. Such phenomenon is continuous to happen upto critical radius of insulation beyond which any insulation wrapped shall decrease the H.T. rate.

* Practical Application

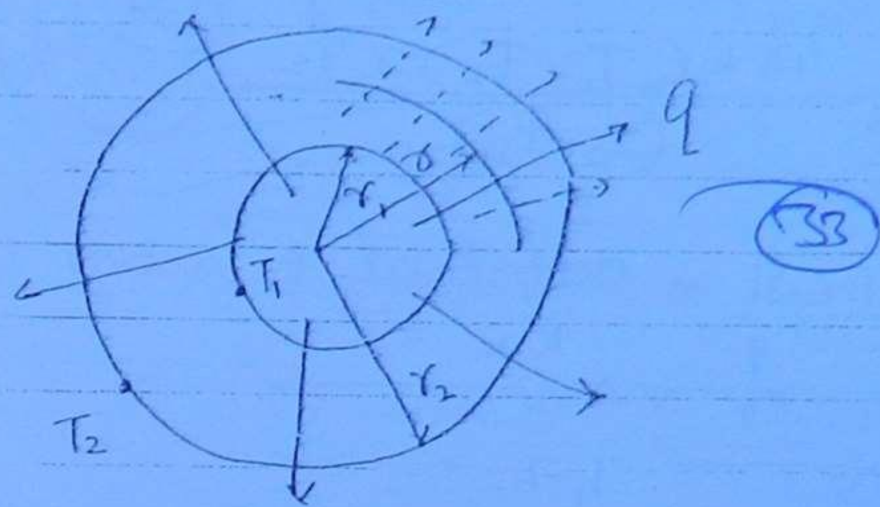
(i) Electric power transmission cables

(ii) Electronic Devices

For spherical cases the critical radius of insulation is

$$\boxed{= \left(\frac{2k}{h}\right)}$$

* Conduction H.T. through a hollow sphere.



The inside surface of hollow sphere is at T_1 i.e. at $r = r_1$

The outside surface of sphere is at T_2 i.e. at $r = r_2$

H.T. occurs radially outwards.

At any radius r, conduction area of HT $(A) = 4\pi r^2$

Fourier law of conduction.

$$q = -KA\frac{dT}{dr}$$

Assumptions :- (i) Steady state H.T. (ii) One dimensional radial H.T. (iii) K is uniform.

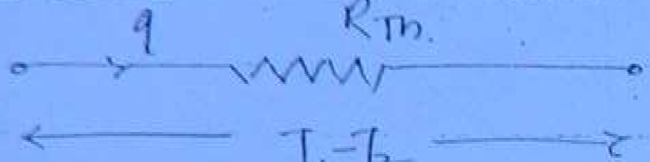
Now $q = -K\,4\pi r^2 \left(\frac{dT}{dr}\right)$ $\qquad T \neq f(\text{time})$

$$\int_{r_1}^{r_2} q \cdot \frac{1}{r^2}\,dr = \int_{T_1}^{T_2} -K4\pi\,dT \qquad q \neq f(r)$$

$$q \cdot \left[\frac{-1}{r}\right]_{r_1}^{r_2} = 4\pi k (T_1 - T_2)$$

$$\therefore q = \frac{4\pi k (T_1 - T_2) r_1 r_2}{(r_2 - r_1)} \quad (34)$$

Thermal Circuit –

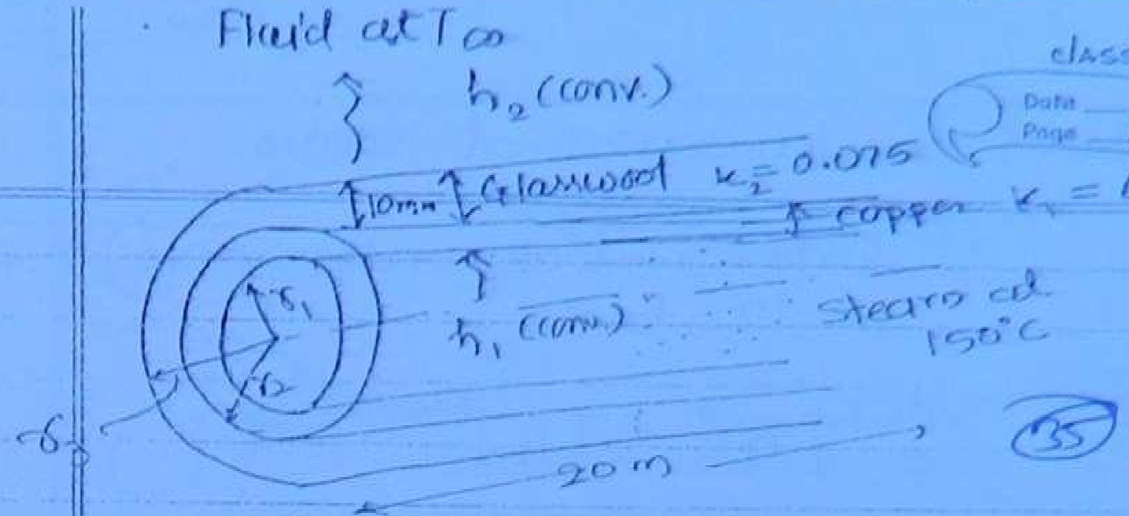


$$R_{Th.} = \frac{q}{T_1 - T_2}$$

$$= \left(\frac{r_2 - r_1}{4\pi k r_1 r_2}\right) \text{ K/Watt}$$

Problem

A copper tube of 20 mm outer dia., 1 mm thickness & 20 m long (thermal conductivity k = 400 W/mK) is carrying saturated steam at 150°C (Convective h.t. coeff. h = 150 W/m²K). The tube is exposed to an ambient temp. of 27°C. The convective h.t. coeff. of air is 5 W/m²K. Glasswood is used for insulation ($k_{Glasswool}$ = 0.075 W/mK). If the thickness of the insulation used is 5 mm higher than the critical thickness of insulation, calculate the rate of heat lost by the steam & the rate of steam condensation in kg/hr. The enthalpy of condensation of steam = 2230 kJ/kg



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(i) Critical Radius of Insulation $= \dfrac{k}{h_{(outside)}}$

$$= \frac{0.075}{5} = 0.015\,m$$

$$= 15\ mm$$

Critical thickness of Insulation $= 15 - 10$
$= 5\ mm$

Actual thickness of Insulation $= 5 + 5$
$= 10\ mm$

$r_1 = 9\ mm \quad r_2 = 10\ mm$

$r_3 = r_2 + 10 = 20\ mm$

Thermal Circuit

$T_{steam} = 150°c$ $\qquad T_{air} = 27°c$

$$\frac{1}{h_1 2\pi r_1 L} \quad \frac{\ln(r_2/r_1)}{2\pi k_1 L} \quad \frac{\ln(r_3/r_2)}{2\pi k_2 L} \quad \frac{1}{h_2 2\pi r_3 L}$$

$$\therefore q = \frac{150 - 27}{\frac{1}{150 \times 2 \times \pi \times 9 \times 10^{-3} \times 20} + \frac{\ln(10/9)}{2\pi \times 400 \times 20} + \frac{\ln(20/10)}{2\pi \times 0.075 \times 20} + \frac{1}{5 \times 2\pi \times 20 \times 10^{-3} \times 20}}$$

$\therefore q = 776$ Watts.

∴ Rate of condensation of steam

$$= \frac{q\ (\text{Watts})}{\text{Latent heat of condensation of steam in J/kg}}$$

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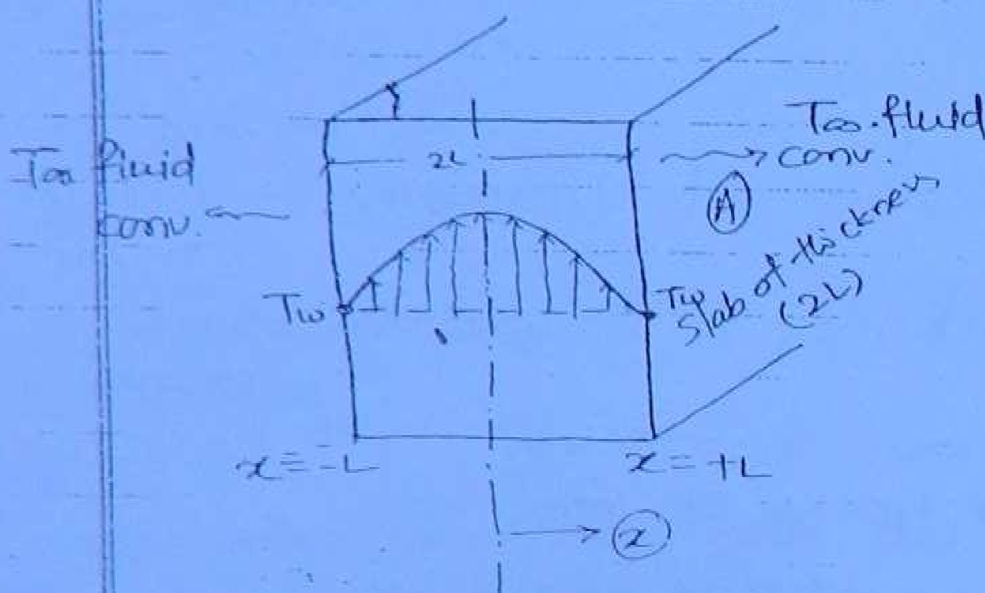
$$= 1.25\ \text{kg/hr.}$$

+ Conduction with Heat Generator in Slab.

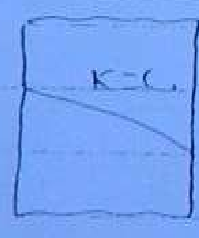


(i) Steady state H.T. Temp $\neq f(\text{Time})$

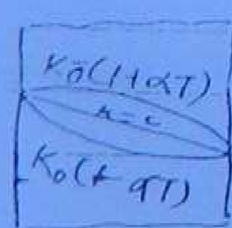
(ii) One dimensional conduction

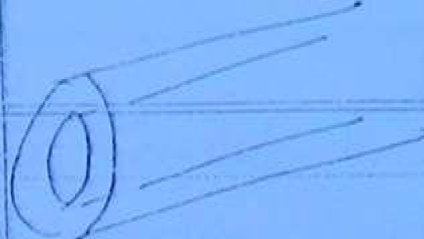
(iii) Uniform heat generation $\dot{q} \neq f(x, y, z)$

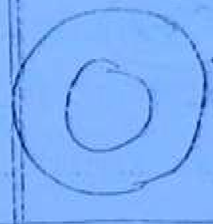
(i) K = const.



(2) K not const



(3)  k = logarithmic

(4)  ← sphere k = hyperbolic (37)

Now we have,

$$\frac{\partial^2 T}{\partial x^2} + \underbrace{\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}}_{\text{zero as one dimensional H.T.}} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\underbrace{\frac{\partial T}{\partial \tau}}_{\text{Steady state, so zero}}$$

$$\therefore \frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{k} = 0$$

$$\frac{d^2 T}{dx^2} = -\frac{\dot{q}}{k}$$

Integrating, $\frac{dT}{dx} = -\frac{\dot{q}}{k}x + C_1$ — (1)

Integrating, $T = -\frac{\dot{q}}{k}\cdot\frac{x^2}{2} + C_1 x + C_2$ — (2)

C_1 & C_2 are const. of integration that are to be evaluated from boundary conditions.

One typical B.C. is at $x = \pm L$
$T = T_w$

This will be possible only when both sides of slab are subjected to the same fluid at the same temp. with the same ~~can~~ h (W/m^2K)

To satisfy this $\boxed{C_1 = 0}$

Let T_0 = Centre line temp. of slab at $x=0$

When T is maximum $\frac{dT}{dx} = 0$ (38)

$\therefore 0 = -\frac{\dot{q}}{k} x \Rightarrow x = 0$

$\therefore$ Max temp at the centre line.

Hence $T_0 = T_{max}$.

Put $x = 0$

$$T = -\frac{\dot{q}}{k} \frac{x^2}{2} + C_2$$

$\therefore x=0 \quad T = T_0 \; ; \quad \boxed{C_2 = T_0}$

The final temperature distribution

$$T = -\frac{\dot{q}}{k} \cdot \frac{x^2}{2} + T_0$$

$$\therefore \boxed{T_0 - T = \frac{\dot{q}}{k} \frac{x^2}{2}} \quad \text{Eqn of Parabola} \quad \text{—— (3)}$$

Put $x = L$

$$\boxed{T_0 - T_w = \frac{\dot{q}}{k} \frac{L^2}{2}} \quad \text{—— (4)}$$

(3)/(4) We get

$$\boxed{\frac{T_0 - T}{T_0 - T_w} = \left(\frac{x}{L}\right)^2} \leftarrow \text{In non dimensional form}$$

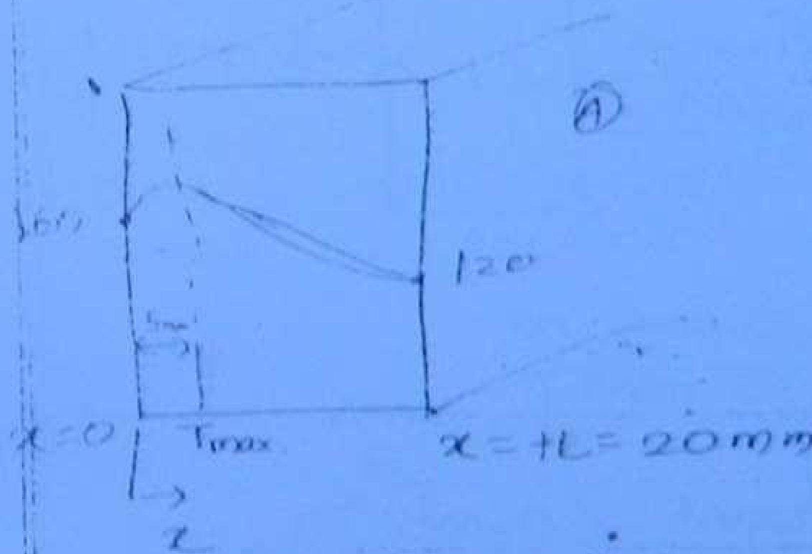
GATE 2007

Consider steady one-dimensional heat flow in a plate of 20 mm thickness with a uniform heat generation of 80 MWatt/m³, the left & right faces are kept at const temp of 160°C & 120°C respectively. The plate has const. k of 200 W/mK. The location of max. temp within plate from its left face is

(i) 15 mm (ii) 10 mm (iii) 5 mm (iv) 0 mm

The max temp. within the plate in °C is

(i) 160 (ii) 165 (iii) 200 (iv) 250



We have,

$$\frac{d^2T}{dx^2}+\frac{\dot q}{k}=0$$

$$\frac{d^2T}{dx^2}=-\frac{\dot q}{k}$$

$\int$ ing

$$\frac{dT}{dx}=-\frac{\dot q}{k}x+C_1$$

Again $\int$

$$T=-\frac{\dot q}{k}\frac{x^2}{2}+C_1x+C_2$$

at $x=0 \quad T = 160$

~~at x=~~

$$\therefore \quad 160 = 0 + 0 + C_2$$

$$\therefore \quad C_2 = 160$$

At $\quad x = 0.02\,m \qquad T = 120^\circ$

$$120 = -\frac{80\times10^6}{200}\left(\frac{0.02^2}{2}\right) + C_1(0.02) + 160$$

$$\therefore \quad C_1 = 2000$$

$\therefore$ Now to get location for max temp

$$\frac{dT}{dx} = 0$$

$$0 = -\frac{80\times10^6}{200}x + 2000$$

$$x = \frac{-2000\times200}{80\times10^6}$$

$$x = 0.005\ m$$

$$\boxed{x = 5\ mm}$$

$$T_{max} = -\frac{80\times10^6}{200}\left(\frac{0.005^2}{2}\right) +$$

$$2000(0.005) + 160.$$

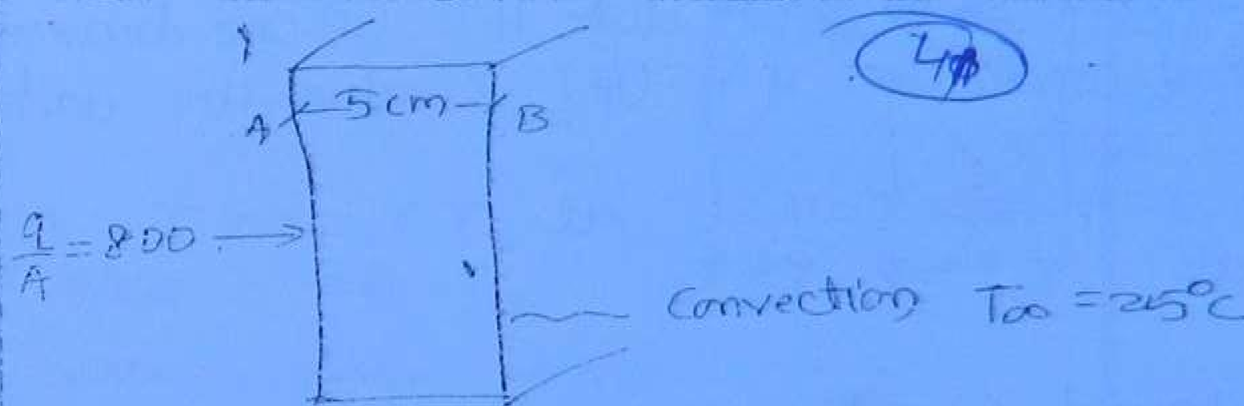
$$\therefore \boxed{T_{max} = 165^\circ C.}$$

IES 99

A steel plate of thickness 5 cm & $k = 20$ W/mK is subjected to a uniform heat flux of 800 W/m² on one surface A and transfers heat by convection with a $h = 80$ W/m²K from the other surface B into the ambient air at $T_\infty = 25°C$. The temperature of surface B transferring heat by convection is ⇒

(i) 25°C (ii) 31°C (iii) 45°C (iv) 55°C.

4



$$\xrightarrow{q/A} \quad T_A \circ \text{---} \underset{b/k}{\text{wwww}} \text{---} \overset{T_B}{\bullet} \text{---} \underset{1/h}{\text{wwww}} \text{---} \circ \quad T_\infty = 25$$

$$\frac{q}{A} = \frac{T_B - T_\infty}{1/h} =$$

$$800 = \frac{T_B - 25}{1/80} \qquad \therefore \boxed{T_B = 35°C}$$

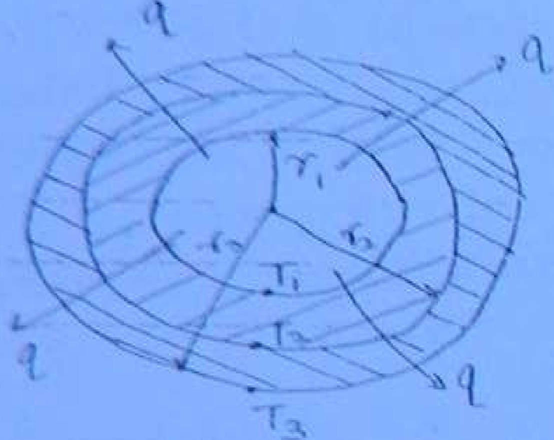
like

$$800 = \frac{T_A - T_B}{b/k} \qquad T_A = ?$$

Prandtl No. is property of fluid.

15th Jan '10, 8:30 AM.

* Conduction H.T. through a composite hollow sphere *



$T_1 > T_2 > T_3$

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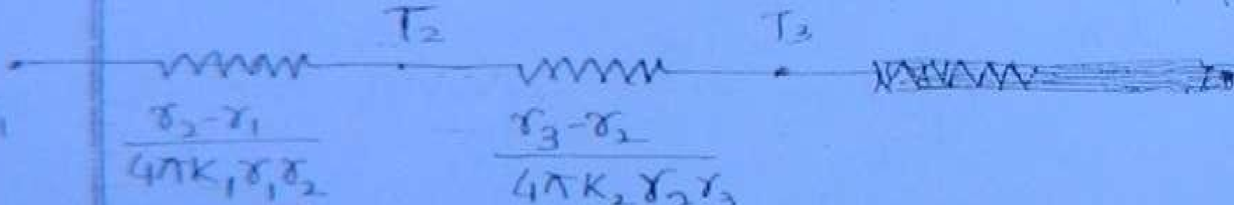
Assumptions ⇒ (i) steady state H.T. (ii) One dimensional Radial conduction H.T. (iii) K values are uniform

At $r = r_1 \Rightarrow T = T_1$ at $r = r_2 \Rightarrow T = T_2$
& at $r = r_3 \Rightarrow T = T_3$

Thermal Circuit



$$q = \frac{T_1 - T_3}{\Sigma R_{Th}}$$

$$q = \left\{ \frac{T_1 - T_3}{\left[\frac{r_2 - r_1}{4\pi K_1 r_1 r_2} + \frac{r_3 - r_2}{4\pi K_2 r_2 r_3} \right]} \right\} \text{ Watts}$$

The Junction temp. i.e. T_2 is given.

$$q = \frac{T_1 - T_2}{r_2 - r_1 / \ldots}$$

$$\therefore T_2 = \cancel{\frac{q\,4\pi K r_1 r_2}{}} - \frac{q(r_2 - r_1)}{4\pi K r_1 r_2} + T_1$$

$$\therefore T_2 = T_1 - \frac{q(r_2 - r_1)}{4\pi K r_1 r_2}$$

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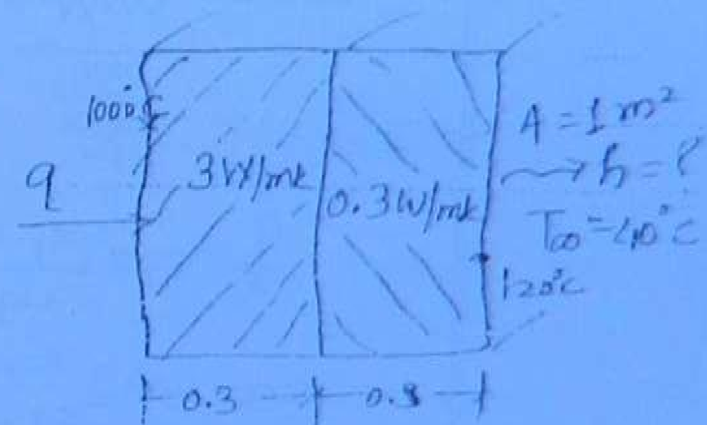
IES Problem

A furnace wall is constructed as shown in the given fig. The heat transfer coeff across the outer surface will be ⇒

(i)

(ii)



$\frac{q}{A}$ 1000°C b/K b/K 120°C   40°C

$\left(\frac{0.3}{3}\right)$ $\left(\frac{0.3}{0.3}\right)$ $(1/h)$

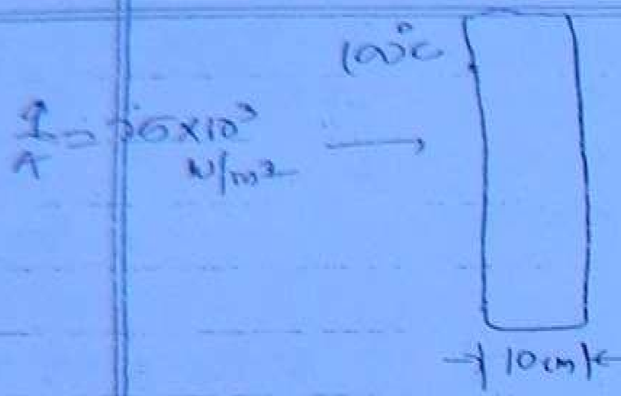
$$\therefore \frac{q}{A} = \frac{1000 - 120}{\left(\frac{0.3}{3}\right) + \left(\frac{0.3}{0.3}\right)} = \frac{120 - 40}{(1/h)}$$

$$\therefore \boxed{h = 10 \quad W/m^2K}$$ (Free Convection in air)

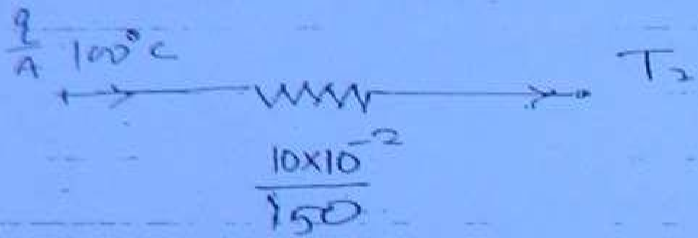
IES 2009

A steel plate of $k = 50$ W/mk & thickness 10 cm passes a heat flux by conduction of 25 kW/m². If the temp of hot surface of plate is 100°C Then what is the temp of the cooler side of the plate.

$k = 50 \ W/mK$



99



$$\therefore \frac{q}{A} = \frac{T_1 - T_2}{b/k}$$

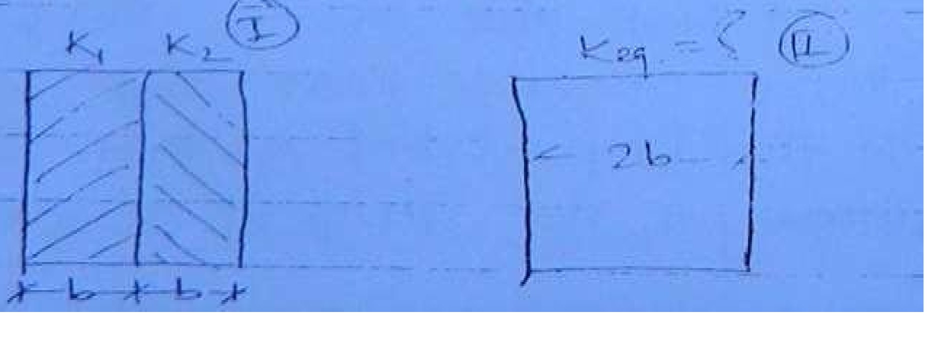
$$\therefore 25\times10^3 = \frac{100 - T_2}{\frac{10\times10^{-2}}{50}}$$

$$\therefore T_2 = 50^\circ C$$

$\boxed{\text{ES 2005}}$

A composite slab has two layers of diff. materials having thermal conductivities K_1 & K_2. If each layer has the same thickness, then what is the equivalent thermal conductivity of the slab.

(a) $\frac{K_1K_2}{2(K_1+K_2)}$ (b) $\frac{2K_1K_2}{(K_1+K_2)}$ ✓ (c) $\frac{(K_1+K_2)}{2K_1K_2}$ (d) $\frac{2(K_1+K_2)}{K_1K_2}$



In case I

$$R_{Th_{(I)}} = \frac{b}{K_1} + \frac{b}{K_2}$$

& In equivalent (II) case.

$$R_{Th_{(II)}} = \frac{2b}{K_{eq.}}$$

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$$\therefore R_{Th_{(I)}} = R_{Th_{(II)}}$$

$$\frac{\cancel{b}}{K_1} + \frac{\cancel{b}}{K_2} = \frac{2\cancel{b}}{K_{eq}}$$

$$\therefore \frac{K_1 + K_2}{K_1 \cdot K_2} = 2/K_{eq}$$

$$\therefore K_{eq.} = \frac{2 K_1 \cdot K_2}{K_1 + K_2}$$

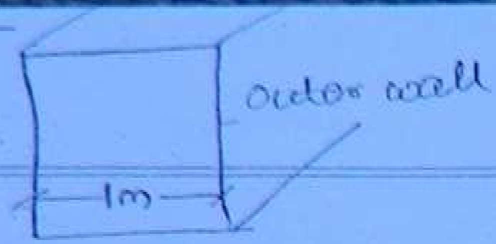
$\boxed{\text{ES 2009}}$

A large concrete slab 1 m thick has one dimensional temp distribution $T = 4 - 10x + 20x^2 + 10x^3$ where T is temp. & x is distance from one face towards the other face of wall. If the slab material has thermal diffusivity of (α) 2×10^{-3} m^2/hr What is the rate of change of temp. at the outer face of the wall.

(a) 0.1 °C/hr (b) 0.2°C/hr (c) 0.3°C/hr (d) 0.4°C/hr

→ We have

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k} = \frac{1}{\alpha}\left(\frac{\partial T}{\partial \tau}\right)$$



(46)

We have for the condition.

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\left(\frac{\partial T}{\partial \tau}\right)$$

$$\therefore \frac{\partial T}{\partial x} = -10 + 40x + 30x^2$$

$$\therefore \frac{\partial^2 T}{\partial x^2} = 60x + 40$$

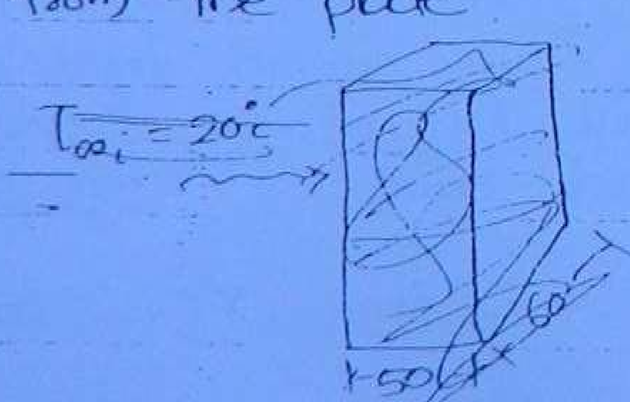
$$\therefore \left(\frac{\partial^2 T}{\partial x^2}\right)_{at\ x=1m} = \frac{1}{\alpha}\left(\frac{\partial T}{\partial \tau}\right)_{at\ x=1.}$$

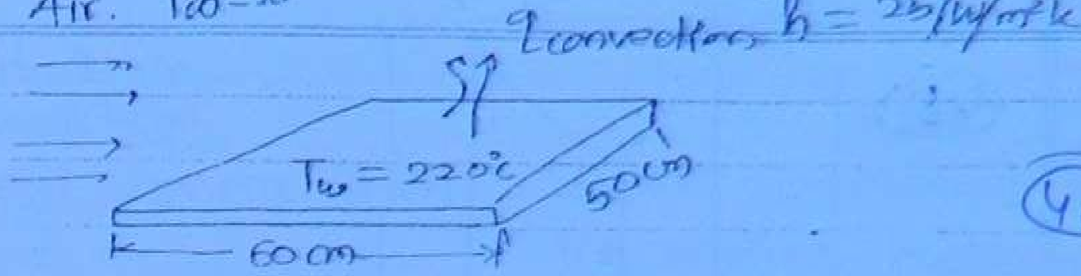
$$\therefore \left(\frac{\partial T}{\partial \tau}\right) = 2\times10^{-3}(60+40)$$

$$\therefore \frac{\partial T}{\partial \tau} = 0.2\ °C/hr$$

$\boxed{ES\ 09}$

An air at 20°C ~~closed~~ blows over a hot plate of 50×60 cm, made of carbon steel maintained at 220 °C. The $h = 25\ W/m^2K$. What will be the heat loss from the plate





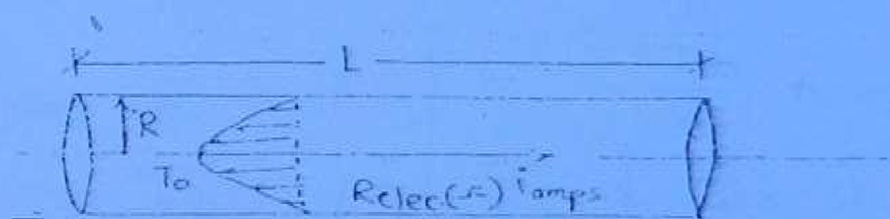
(46)

$\therefore$ Rate of H.T. (from top as well as bottom) $= hA(T_w - T_\infty)$

$$= 25 \times \left(\frac{50}{100} \times \frac{60}{100}\right) \times 2[220 - 20]$$

$$= 3000 \text{ Watts}$$

* Heat Generation in the cylinder :-



Heat generated per second $= Vi$

$= i^2 R_{ele}$ Watts

Assumptions $\Rightarrow$ i) Steady state heat transfer $T \neq f(\text{time})$
(ii) The heat generated in wire should be taken/given to the surrounding fluid at T_∞ by convection for which
One dimensional radial conduction H.T. $(T = f(r))$

The generalised heat conduction eq^n in cylindrical co-ordinates is

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

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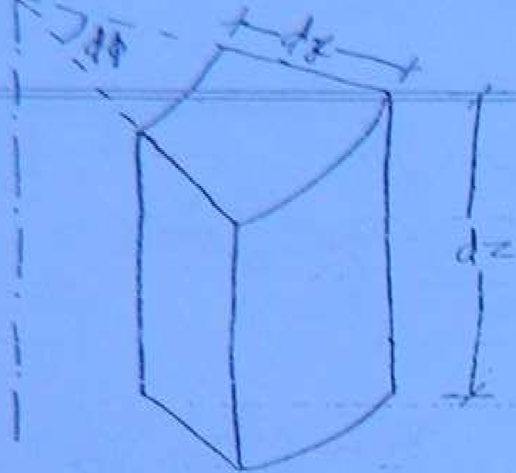

r = Radial
ϕ = Azimuthal
z = Axial

Since temperature is function of radius only
i.e. $T = f$ (Radial Dirn Only)

$$\therefore \frac{\partial^2 T}{\partial \phi^2} = 0 \; ; \; \frac{\partial^2 T}{\partial z^2} = 0 \; ; \; \frac{\partial T}{\partial \tau} = 0 \text{ (Steady State)}$$

$$\Rightarrow \frac{d^2 T}{dr^2} + \frac{1}{r}\frac{dT}{dr} + \frac{\dot{q}}{k} = 0$$

$$r \cdot \frac{d^2 T}{dr^2} + \frac{dT}{dr} = -\frac{\dot{q}}{k} r$$

As we have $\Rightarrow \frac{d}{dr}\left(r \cdot \frac{dT}{dr}\right) = \left[r\frac{d^2T}{dr^2} + \frac{dr}{dr}\frac{dT}{dr}\right]$

$$\frac{d}{dr}\left(r \cdot \frac{dT}{dr}\right) = -\frac{\dot{q}}{k} r$$

Integrating

$$r \cdot \frac{dT}{dr} = -\frac{\dot{q}}{2k} r^2 + C_1$$

$$\frac{dT}{dr} = -\frac{\dot{q} r}{2k} + \frac{C_1}{r} \quad \text{———} (i)$$

Integrating

$$\boxed{T = -\frac{\dot{q} r^2}{4k} + C_1 \log r + C_2} \quad \text{———} (ii)$$

where C_1 & C_2 are constants of integration to be obtained from B.C.

should be ∞. $\therefore C_1 = 0$]

(49)

Boundary condition

(i) The heat generated in the wire = Heat conducted at the surface radially.

$$\dot{q}(\pi r^2 L) = -k\, 2\pi r L \left(\frac{dT}{dr}\right)_{at\ r=R}$$

$$\therefore \left(\frac{dT}{dr}\right)_{at\ r=R} = -\frac{\dot{q}R}{2k} \quad \text{--- (iii)}$$

According to (1)

$$\left(\frac{dT}{dr}\right)_{r=R} = -\frac{\dot{q}R}{2k} + \frac{C_1}{R} \quad \text{--- (iv)}$$

from (iii) & (iv)

$$-\frac{\dot{q}R}{2k} = \frac{-\dot{q}R}{2k} + \frac{C_1}{R}$$

$$\therefore \boxed{C_1 = 0}$$

(ii) At the centre line axis i.e. at $r=0$, $T=T_0$
(T_0 = Centre line Temp.)

When T is maximum,

$$\frac{dT}{dr} = 0 \Rightarrow 0 = \frac{-\dot{q}r}{2k} + 0$$

$$\Rightarrow r = 0$$

$\therefore$ T_0 is T_{max} i.e. temp. at the axis

from eqn (ii)

Now $T = -\dfrac{\dot{q}r^2}{4k} + C_2$

To get, Put $r=0 \Rightarrow T = T_0 = C_2$

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$$\therefore \quad T = \frac{-\dot{q}r^2}{4k} + T_0$$

$$\boxed{T_0 - T = \frac{\dot{q}r^2}{4k}}$$

This is the eqn of temp. distribution which is of parabolic nature.

Let T_w = Surface temp of wire (i.e. at $r=R$)

Put $r=R \Rightarrow T_0 - T_w = \dfrac{\dot{q}R^2}{4k}$

$$\Rightarrow \frac{T_0 - T}{T_0 - T_w} = \left(\frac{r}{R}\right)^2$$

To get T_w from energy balance, for steady state conditions $\Rightarrow$

Total heat generated in the wire = Heat convected at the surface

$$\dot{q}(\cancel{\pi}R^{\cancel{2}}\cancel{L}) = h(2\cancel{\pi}\cancel{R}\cancel{L})(T_w - T_\infty)$$

$$\therefore \quad \frac{\dot{q}R}{2h} = T_w - T_\infty$$

$$\therefore \quad T_w = T_\infty + \frac{\dot{q}R}{2h}$$

Centre line Temp — $\boxed{T_0 = T_w + \dfrac{\dot{q}R^2}{4k}}$

A current of 200 Amps is passed through a stainless steel wire 0.25 cm in dia. The resistivity of steel is 70 $\mu\Omega$cm & length of wire is 1 m. If the outer surface temp. of wire is maintained at 180°C. Calculate the centre temp. Assume k of steel as 30 W/mK.

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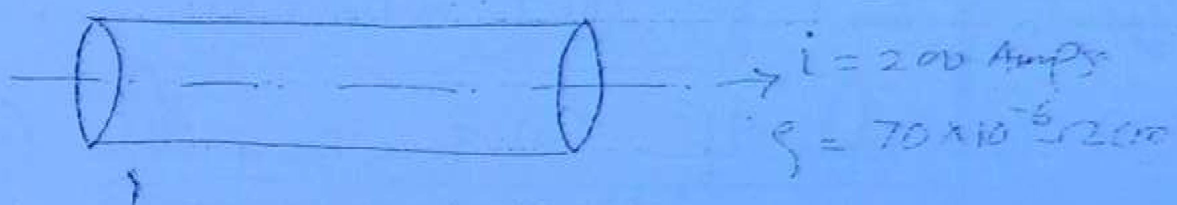


$$R_{elect.} = \rho \frac{L}{A} \Rightarrow R_{elec.} = \frac{70 \times 10^{-6}}{100} \times \frac{1}{\frac{\pi}{4}\left(\frac{0.25}{100}\right)^2} =$$

$$= \frac{70 \times 10^{-4}}{4.908 \times 10^{-6}}$$

$$= \cancel{14.262} \quad 14.26 \times 10^{-2}$$

$$\cancel{1.426 \times 10^{-3}}$$

Heat Generated per unit volume

$$\dot{q} = \frac{i^2 R_{elect}}{(\pi R^2 L)} \text{ Watts}/m^3$$

$(\pi R^2 L) \leftarrow$ Volume of wire

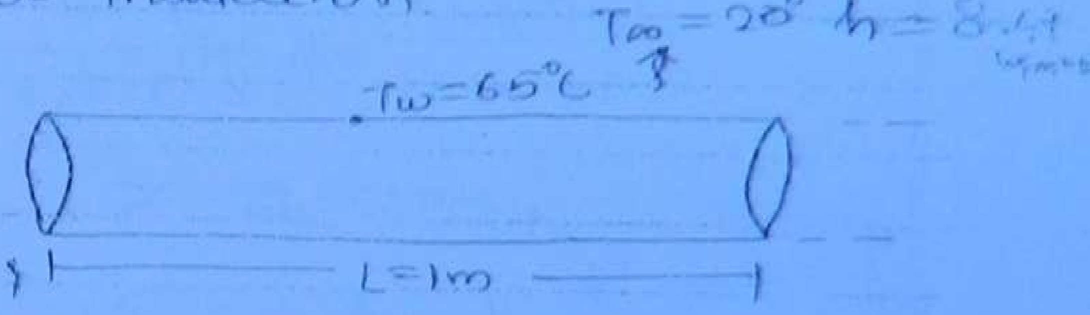
$$\therefore \dot{q} = \left[\frac{200^2 \times 0.143}{\pi \left[\frac{0.25}{2 \times 100}\right]^2 \times 1}\right] \; W/m^3$$

$$\therefore \dot{q} = 1.165 \times 10^9 \; W/m^3$$

$$T_0 = T_w + \frac{\dot{q} R^2}{4k} = 180 + \frac{(\dot{q})\left(\frac{0.25}{2 \times 100}\right)^2}{4 \times 30}$$

$$T_0 = 195.2°C$$

A 10 mm cable is to be laid in a atmosphere of 20°c ($h = 8.49\ W/m^2K$). The surface temp of the cable is likely to be 65°C due to heat generated within the wire. Find the rate of heat loss a) with critical radius of insulation
b) Without insulation



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$q_{\text{Without Insulation}} = h A (T_w - T_\infty)$

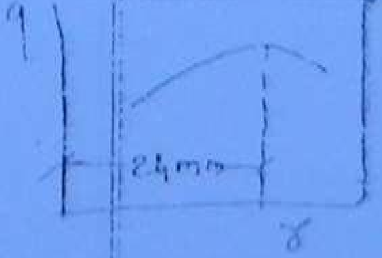
$q = 8.49 \times (\pi \times \frac{10}{100} \times 1)(65 - 20)$

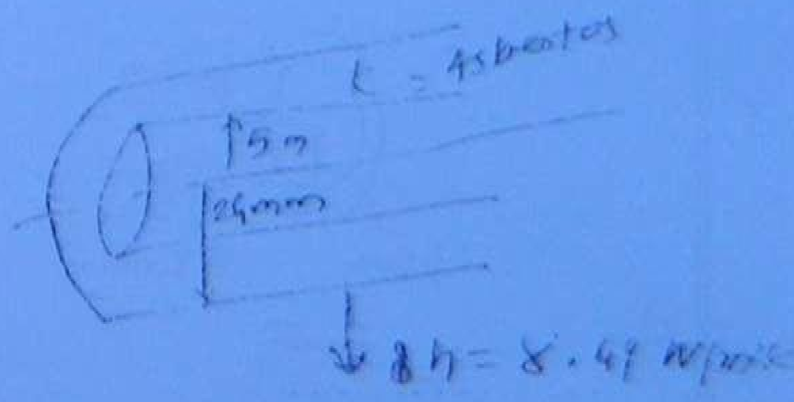
$q = 12.09\ W/m$

Critical radius of insulation (Asbestos) $= \frac{k}{h}$

$= \frac{0.2}{8.49}$

$= 0.024\ m = 24\ mm$





$$65^\circ C \longrightarrow \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi K L} \longrightarrow \frac{1}{h\,2\pi r_2 L} \longrightarrow 20^\circ C$$

(54)

$$q_{\text{with Ins.}} = \frac{65-20}{\frac{\ln(24/5)}{2\pi \times 0.2 \times 1} + \frac{1}{8.49 \times 2 \times \pi \times \frac{24}{1000} \times 1}}$$

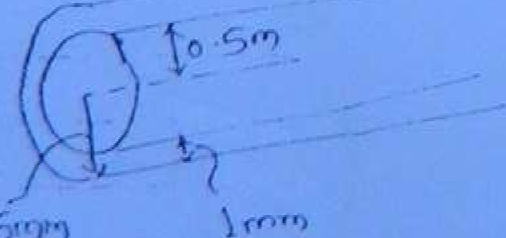
$$q_{\text{with Insu.}} = 19 \text{ W}$$

A copper wire of radius 0.5 mm is insulated with a thickness of 1 mm having a thermal conductivity of 0.5 W/m K. The outside surface convective H.T. coeff is 10 W/m²K. If the thickness of insulation is raised by 10 mm, then the electrical current carrying capacity of wire will a

[a] Increase [b] Decrease [c] Remains Same

[d] depends upon the electrical conductivity of wire.

$\Longrightarrow$



$$r_{critical} = \frac{k}{h} = \frac{0.5}{10} = 50 \text{ mm}$$

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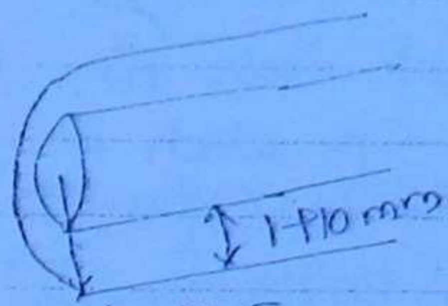


$r_{critical} = 50$ mm

$\therefore r_2 = 1.5$ mm &

$r_2' = 11.5$ mm

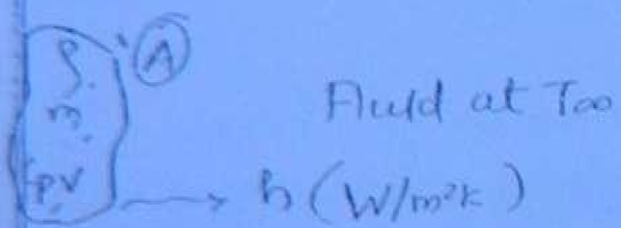
$\therefore$ As radius is increasing heat loss will increase & current carrying capacity will also increase.

* Unsteady State or Transient heat conduction

In this type of conduction H.T., temp. of the body may change w.r.t time

Consider a body of mass m, density ρ, volume V, Specific heat C_p which is at an initial temp. of T_i (When taken out of the furnace) is suddenly exposed to an environment at T_∞. Since the body looses heat by convection to the fluid of environment with a convection heat transfer coeff. of h W/m^2K) the temp. of body keeps on decreasing as the time progresses

Area of contact (A)



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Let T_i = Initial temp. of body at an instance of time $\tau = 0$ sec.

Let T = Temp. of body at any instance of time τ sec, measured from the instance when the body is just exposed to environment

$T = f(\tau)$

Writing the energy balance at any instance of time τ sec.

The rate of convection heat loss from surface of body to fluid = Rate of decrease in internal energy of the body

Temp w.r.t. Time

$$hA(T-T_\infty) = -mc_p\left(\frac{dT}{d\tau}\right)$$

$$= -\rho V c_p\left(\frac{dT}{d\tau}\right)$$

Keeping all other parameter const. & separating the variable i.e. Temp. (T) & Time (τ)

$$-\frac{hA}{\rho V c_p}\,d\tau = \frac{dT}{(T-T_\infty)}$$

$$-\left(\frac{hA}{\rho V c_p}\right)d\tau = \frac{dT}{(T-T_\infty)}$$

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Integrating.

$$\int_0^\tau -\left(\frac{hA}{\rho V c_p}\right)d\tau = \int_{T_i}^{T}\frac{dT}{(T-T_\infty)}$$

$$\left(\frac{hA}{\rho V c_p}\right)\tau = \log_e\left(\frac{T_i-T_\infty}{T-T_\infty}\right)$$

$$\boxed{\left(\frac{T_i-T_\infty}{T-T_\infty}\right) = e^{\left(\frac{hA}{\rho V c_p}\right)\tau}}$$

As the time progresses the rate of cooling of body decreases. Since the temp diff b/w the body & fluid decreases.